RELATIVISTIC SOLUTIONS FOR

THE FOLLOWING PHENOMENA :-

(i) The Value of the Velocity of Light.

(ii) The Fitzgerald Length Contraction.

(iii) The Ehrenfest Paradox.

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ABSTRACT.

This paper, via the application of the Special Theory of Relativity, provides a physical and mathematical solution to (i) the value of the velocity of light, (ii) the contraction of the length of a rod in motion in the direction of its length with a relativistic velocity, and (iii) the Ehrenfest Paradox concerning the contraction of the circumference of the perimeter of a spinning disc.

CONTENTS.

1.0. Introduction.

2.0 The Value of the Velocity of Light.

3.0 The Fitzgerald Length Contraction.

4.0 The Ehrenfest Paradox.

5.0 Conclusions.

REFERENCES.

1.0 Introduction.

The three questions/paradoxes that are the subject of this paper are :

(i) Why is the velocity of light the particular value that it is, 2.997924579E10 cm/sec.

(ii) What is the precise cause of the Fitzgerald contraction of a rod in the direction of its length when in motion at a relativistic velocity.

(iii) How does the circumference of a disc, spinning at a relativistic rate, contract when neither π nor its radius can change. This is the Ehrenfest Paradox.

All three of these questions, particularly the first, have been addressed in the past, but it is believed that none have been provided with a sufficiently definitive answer.

The solutions provided in this paper are all based upon the application of Albert Einstein's Special Theory of Relativity as mathematically reformulated in [1].
For a full appreciation of this paper it is recommended that this reference be read first.

2.0 The Value of the Velocity of Light.

With regard to this question, the two most prominent explanations put forward in the past are:

(a) It is given by the ratio of the Planck length to the Planck Time.

(b) It is the root of the inverse product of the magnetic vacuum permeability of free space, and the electric vacuum permittivity of free space.

Concerning (a), the derivation of the Planck length and Planck time both include the velocity of light as a parameter, viz.

$$l_p = \left(\frac{G\hbar}{c^3}\right)^{1/2} \tag{2.1}$$

and

$$t_p = \left(\frac{G\hbar}{c^5}\right)^{1/2} \tag{2.2}$$

where G is Newton's constant of gravitation and $\hbar$ is the reduced Planck constant. Clearly the ratio of l_p to t_p produces a simple equation for c,

$$\frac{l_p}{t_p} = c \tag{2.3}$$

However, Planck defined both l_p and t_p from just a dimensional analysis of the three constants G, $\hbar$ and c. Consequently, (2.3) cannot be considered a rigorous derivation of the value of the velocity of light.

Concerning (b), the relationship is

$$c = \left(\frac{1}{\mu_0 \varepsilon_0} \right)^{1/2} \tag{2.4}$$

Where μ_0 is the magnetic vacuum permeability of free space and ε_0 the electric vacuum permittivity. This relationship can be derived from Maxwell's electromagnetic theory. However, while it is a rigorously derived relationship, it merely changes the question as to why do μ_0 and ε_0 have the values that they do. In any case some theorists prefer to restate (2.4) as a definition of ε_0. i.e.

$$\varepsilon_0 = \frac{1}{\mu_0 c^2} \tag{2.5}$$

Furthermore, theorists now define μ_0 in terms of α, the fine structure constant, which itself has no rigorous mathematical derivation.
Consequently, the approach proposed by (b) merely shifts the question around in a circle which therefore eludes a definitive answer.

The approach adopted here, was to some extent addressed in [2], but did not specifically identify the reason for the value of *c*. To do that now, Fig D.1 is repeated from [2] thus.

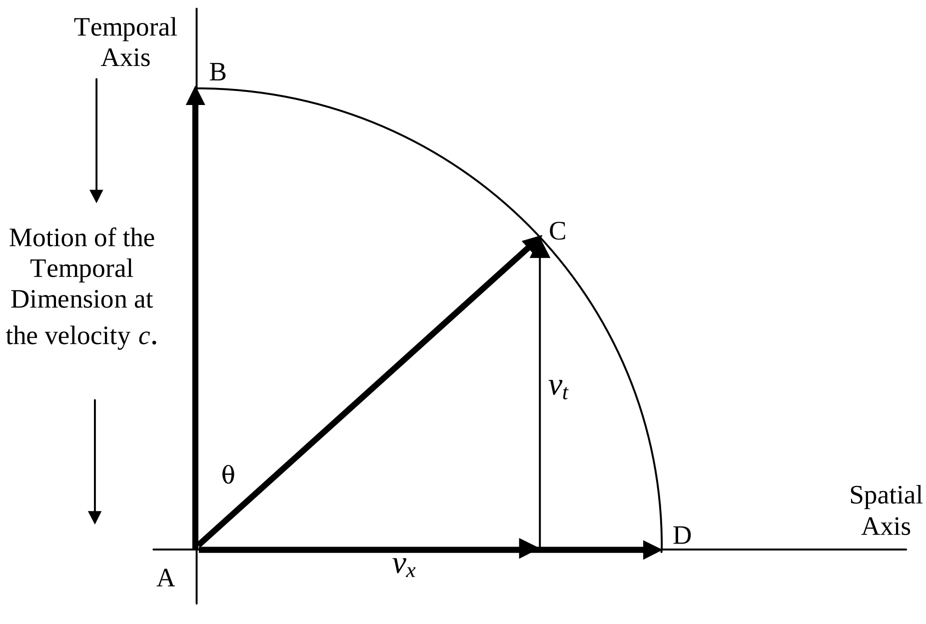

Fig 2.1 – Existence in the Relativistic space-time.

From the theory in [1], if an object at A is stationary, its Existence Velocity Vector lies along the line A $\longrightarrow$ B, and is entirely due to the motion of the temporal dimension through the spatial domain at the velocity *c*.
If the object at A possess a spatial velocity, its Existence Velocity Vector rotates by the appropriate angle θ in the spatial – temporal plane, to lie along the line A$\longrightarrow$ C. Its magnitude is still *c* because as shown in [1], the magnitude of any Existence Velocity can only have the single constant value of *c*. However, it is now made up of its spatial velocity

$$v_x = c\sin\theta \qquad (2.6)$$

and its temporal velocity which is reduced to

$$v_t = c\cos\theta = c\left(1 - \frac{v_x^2}{c^2}\right)^{1/2} \qquad (2.7)$$

This results in a reduction in the rate of passage of time for the object in motion and an increase in its mass.
If the object at A possess a spatial velocity of *c*, then its Existence Velocity Vector lies along the line A→ D and its temporal velocity is zero. Hence under this scenario the rate of passage of time for the object is zero. The only object capable of this existence is the photon which has zero rest mass.
This scenario clearly shows that the velocity of a photon has the constant velocity possessed by *c*, because its Existence Velocity Vectors origin is the same as that with which the temporal dimension passes through the three spatial dimensions. It was also shown in [1], that this is also a physical explanation for the existence of time in the spatial domain.

3.0 The Fitgerald Contraction of Length.

Again this subject was to some extent covered in [3], but without a specific reference to this apparent phenomena. Repeating a version of Fig. 2.1 above incorporating a moving rod.

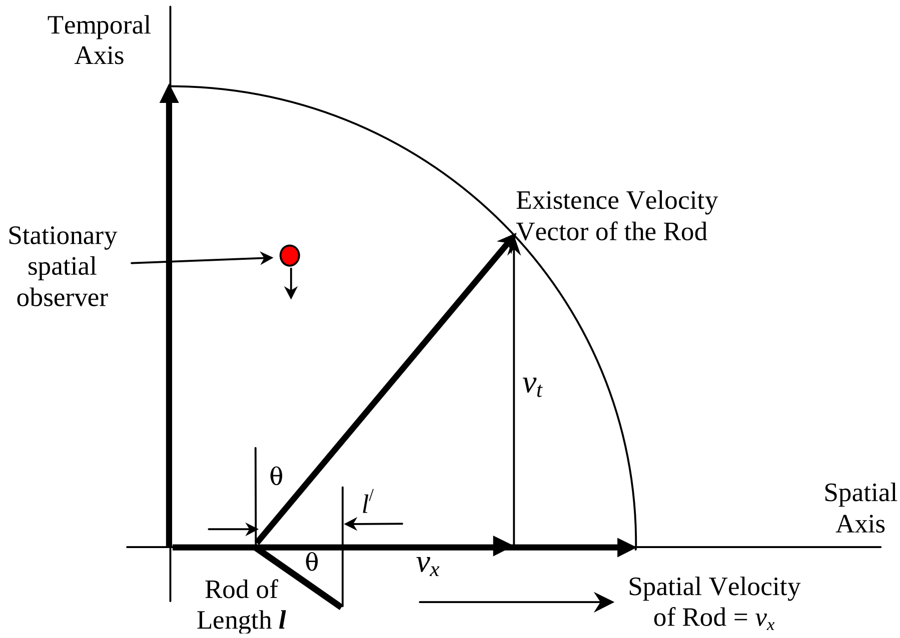

Fig 3.1 – Rod in Motion Along the Three Dimensional Spatial Axis with Velocity v_x.

As the rod moves along the three dimensional spatial axis, its Existence Velocity Vector rotates into the spatial-temporal domain as shown in Fig.3.1. The physical existence of the rod, in remaining orthogonal to its Existence Velocity Vector therefore 'rotates' into the temporal dimension. Consequently a stationary observer in the spatial domain would observe the length of the rod as

$$l' = l\cos\theta = l\left(1 - \frac{v_x^2}{c^2}\right)^{1/2} \tag{3.1}$$

while an observer travelling with the rod would still see its correct length *l*. In the same way the rod would experience time dilatation due to its reduced temporal velocity as given by (2.7). It is clear therefore that the rod does not actually undergo a physical change in length.

4.0 The Ehrenfest Paradox.

Paul Ehrenfest proposed his paradox in 1909 as to how could the circumference of a disc, spinning at a relativistic angular rate, contract when neither π, a fundamental constant, nor *R*, the radius of the disc, be affected by the motion, the latter because it is orthogonal to the direction of the motion of the periphery. The solution is similar to that for the Fitzgerald length contraction but, just marginally more complex. Consider Fig. 4.1 below.

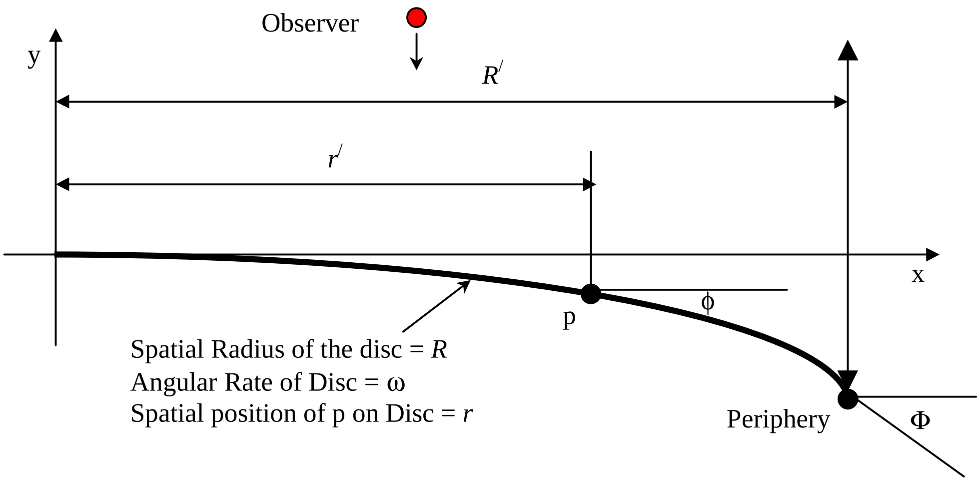

Fig. 4.1 – Configuration of a Spinning Disc in Space -Time.

In Fig. 4.1 the y axis doubles as the axis of rotation and the temporal axis.

Note that the disc becomes curved into the temporal dimension because different points along the radius posses different spatial velocities. This does not mean however, that the disc physically distorts in the spatial domain.

The velocity of the disc's periphery is ω*R* and of the point p is ω*r*.

From (2.7) it is clear that in Fig. 4.1

$$\cos\phi = \left(1 - \frac{\omega^2 r^2}{c^2}\right)^{1/2} \tag{4.1}$$

which re-arranges to

$$r = \frac{c}{\omega}\sin\phi \tag{4.2}$$

so that

$$dr = \frac{c}{\omega}\cos\phi\, d\phi \tag{4.3}$$

Therefore from Fig.4.1

$$dr^{/} = \frac{c}{\omega}\cos^2\phi\, d\phi \tag{4.4}$$

Integrating

$$R^{/} = \frac{c}{\omega}\int_0^{\Phi}\cos^2\phi\, d\phi \tag{4.5}$$

which evaluates to

$$R^{/} = \frac{c}{2\omega}\left(\Phi + \frac{1}{2}\sin 2\Phi\right) \tag{4.6}$$

and from Fig. 4.1

$$\Phi = \sin^{-1}\frac{\omega R}{c} \quad \text{and} \quad \sin 2\Phi = \sin 2\sin^{-1}\frac{\omega R}{c}$$

so that $R^{/}$ becomes

$$R^{/} = \frac{c}{2\omega}\left[\sin^{-1}\frac{\omega R}{c} + \frac{1}{2}\sin 2\sin^{-1}\frac{\omega R}{c}\right] \tag{4.7}$$

with the observed circumference given by $2\pi R^{/}$.

When ωR = c, i.e. the periphery is moving at velocity of light, then

$$\sin^{-1}\frac{\omega R}{c} = \sin^{-1}1 = \frac{\pi}{2}$$

and

$$\frac{1}{2}\sin 2\sin^{-1}\frac{\omega R}{c} = \frac{1}{2}\sin\pi = 0$$

so that

$$R^{/} = \frac{\pi R}{4} \tag{4.8}$$

and the observed circumference is then

$$2\pi R^{/} = \frac{\pi^2 R}{2} \tag{4.9}$$

Note that the periphery of the disc can achieve the velocity of light because the remainder of the disc is spinning at a lower velocity, so that while the mass of the disc relativistically increases, it cannot approach an infinite value. Also note that again the circumference of the disc does not undergo a physical contraction.

5.0 Conclusions.

With regard to the value of the velocity of light, it may be said that the solution proposed here has merely shifted the question into the temporal dimension. i.e. why does the velocity of the temporal dimension through the three dimensions of the spatial domain have the particular value that it does, *c*. A perfectly valid question.

However, if so, the important point is that the question has been shifted out of the three dimensional spatial domain. As such a solution cannot therefore be derived from parameters within the spatial domain. Furthermore, the temporal dimension is one part of the four dimensional space-time manifold, and the velocity of its motion through the spatial domain must therefore be considered as a fundamental property of the manifold itself. It is therefore considered very unlikely that the source of the motion, or the reason for its value will ever be determined.

Concerning the Fitzgerald length contraction and the Ehrenfest paradox, the rotation of these objects into the temporal dimension does not mean that they no longer exist in the spatial domain. Nor does it mean that they have 'travelled' in time. The possibility of time travel was disproved in [3]. While the objects remain in the spatial domain, and experience time dilatation due to their reduced temporal velocity, their physical dimensions exist at different points in time in the temporal dimension due to their Existence Velocity Vectors orientation compared to a stationary object.
With particular regard to the rod, this results in an interpretation by a stationary observer as a contraction of length. The concept is perhaps not an easy one to visualise.

With regard to the spinning disc, although the radius is orthogonal to the peripheral motion, due to the variable rotation of it into the temporal dimension, it is in fact the radius that appears to contract thus resulting in an apparent contraction of the circumference.
When, hypothetically, the spatial velocity of these objects reaches light speed, as shown above the apparent circumference of the spinning disc reduces to a minimum. The rod under this condition would fully rotate into the temporal dimension and become effectively invisible to a stationary observer.
It is suggested that this may also be the reason why beams of sunlight and/or starlight cannot be seen traversing space. Light from these objects is only visible when transmitted directly towards an observer. Consequently, light transmitted from the Sun towards the planet Mars for instance, cannot be seen in the intervening space. It can only be seen when it reflects from the planet directly towards an observer. This comment also applies to sunlight traversing space within the Earth's atmosphere.

REFERENCES.

[1] P.G.Bass, *The Special Theory of Relativity – A Classical Approach*, www.relativitydomains.com.

[2] P.G.Bass, *Derivation of the Schrodinger, Klein-Gordon and Dirac Equations of Particle Physics by Classical Methods*, www.relativitydomains.com.

[3] P.G.Bass, *An Investigation into Three Relativistic Fringe Subjects*, www.relativitydomains.com.