THE TWIN PARADOX

A RELATIVISTIC DOMAIN RESOLUTION

Peter G.Bass

www.relativitydomains.com

R4 Version 1.0.0
January 2012

ABSTRACT

This short paper shows that the so called "Twin Paradox" of Special Relativity, is in fact not a paradox at all, and is easily resolvable using the Relativistic Domain mathematical formulation of the Special Theory.

CONTENTS

1.0 Introduction.

The "Twin Paradox" is a thought experiment in Special Relativity, in which the ages of twins are compared after one has made a high speed journey over a long period of time, while the other has remained stationary. The proposed paradox arises because each twin sees the other as travelling, and so should apparently, due to the effects of relativistic time dilatation, see the other as having aged more slowly than himself.

Ever since Einstein's demonstration of the presence of time dilatation in the Special Theory, there has been a significant amount of discussion on the "Twin Paradox". Some protagonists maintaining that it represents an unresolved issue, while others hold the belief that it is not a paradox at all, primarily because of the lack of symmetry in the inertial frames of reference of each twin. Accordingly, there have been a considerable number of attempts, some of which are continuing, [2], to demonstrate this, but, to date, none have proved to be entirely satisfactory, [1]. It is believed that this is because such attempts have not fully considered the temporal aspects of the problem, only the spatial.

The analysis in the remainder of this short paper will approach the problem using the Relativistic Domain formulation of the Special Theory, as presented in [3], thereby giving the temporal issues equal prominence with the spatial.

2.0 The Twin Paradox Resolution.

In the ensuing analysis, each twin assumes the role of observer of the other in their separate frames of reference. Thus the stationary twin is designated observer O_S, while the travelling twin, observer O_T.

2.1 The Existence Velocity of Both Twins in Pseudo-Euclidean Space-Time.

The Existence Velocity, as defined in [3], of each twin in the reference frame of O_S is shown below in Fig. 2.1.

(i) The outward journey of O_T.

(ii) The inward journey of O_T.

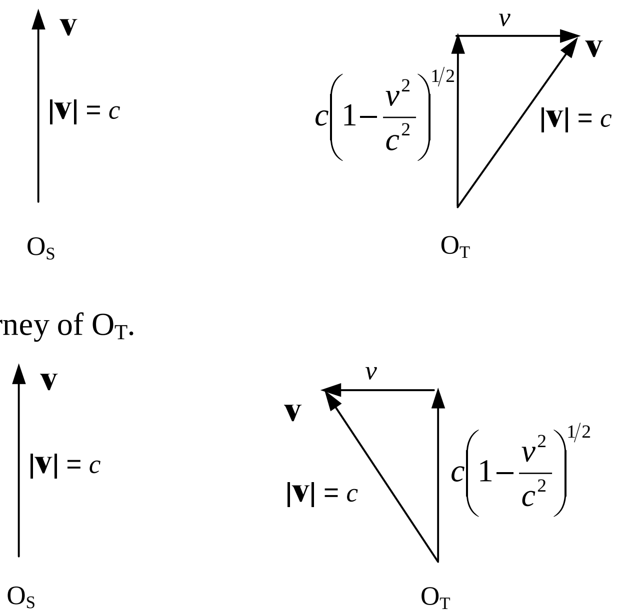

Fig. 2.1 - The Existence Velocities of O_S and O_T in Pseudo-Euclidean Space-Time.

2.2 Assessment of the Spatial Velocity of Each Twin by the Other.

It is important that each observer correctly determines the spatial velocity of the other in his own frame of reference.

(i) The average spatial velocity of O_T as determined by O_S is simply

Outward journey

$$\frac{dx}{dt} = v \tag{2.1}$$

Inward journey

$$\frac{dx}{dt} = -v \tag{2.2}$$

(ii) The average spatial velocity of O_S as determined by O_T is, using the Lorentz transformations between reference frames.

Outward journey

$$dx' = \frac{dx - vdt}{\left(1 - {v^2}/{c^2}\right)^{1/2}} = \frac{-vdt}{\left(1 - {v^2}/{c^2}\right)^{1/2}} \tag{2.3}$$

$$dt' = \frac{dt - dx\, {v}/{c^2}}{\left(1 - {v^2}/{c^2}\right)^{1/2}} = \frac{dt}{\left(1 - {v^2}/{c^2}\right)^{1/2}} \tag{2.4}$$

and therefore

$$\frac{dx'}{dt'} = -v \tag{2.5}$$

Inward journey

$$dx' = \frac{dx + vdt}{\left(1 - {v^2}/{c^2}\right)^{1/2}} = \frac{vdt}{\left(1 - {v^2}/{c^2}\right)^{1/2}} \tag{2.6}$$

$$dt' = \frac{dt + dx\, {v}/{c^2}}{\left(1 - {v^2}/{c^2}\right)^{1/2}} = \frac{dt}{\left(1 - {v^2}/{c^2}\right)^{1/2}} \tag{2.6}$$

and therefore

$$\frac{dx'}{dt'} = +v \tag{2.7}$$

Hence each twin correctly assesses the spatial velocity magitude of the other.

It is important to note that (2.4) and (2.6) is a measure of the length of a unit of time in the reference frame of O_T, and not the journey travel time.

2.3 Assessment of Travel Time, (Age Difference).

Because this is only a thought experiment, the assessment of the temporal component of Existence Velocity of each twin by the other, may be incorporated as a realisable measurement.

(i) The travel time of O_T as determined by O_S.

The travel time of the round trip as measured by O_S is Δt. The temporal velocity of O_T, for both outward and inward journeys is, from Fig. 2.1 given by

$$c' = c\left(1-\frac{v^2}{c^2}\right)^{1/2} \tag{2.8}$$

Therefore the temporal distance travelled by O_T, as determined by O_S is

$$\Delta x_0 = c'\Delta t = c\left(1-\frac{v^2}{c^2}\right)^{1/2}\Delta t \tag{2.9}$$

and consequently, the amount of time that has passed for O_T as assessed by O_S is

$$\frac{\Delta x_0}{c} = \left(1-\frac{v^2}{c^2}\right)^{1/2}\Delta t = \Delta t' \tag{2.10}$$

(ii) The travel time of O_S as determined by O_T.

The time of the round trip as measured by O_T is given by (2.10), i.e.$\Delta t'$. The temporal velocity of O_S as measured by O_T is

$$c = \frac{c'}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \tag{2.11}$$

Thus the temporal distance travelled by O_S, as determined by O_T is

$$\Delta x_0' = \frac{c'}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}\Delta t' \tag{2.12}$$

and therefore the amount of time that has passed for O_S as assessed by O_T is from (2.8) and (2.12)

$$\frac{\Delta x_0'}{c'} = \frac{\Delta t'}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} = \Delta t \tag{2.13}$$

Therefore, by (2.10), O_S assesses that O_T has aged less than himself by the amount (Δt - $\Delta t'$), and conversely, by (2.13), O_T assesses that O_S has aged more than himself by the same amount.

3.0 Assessment of Symmetry.

The degree of symmetry between the reference frames of O_S and O_T can be determined from the above results as follows.

(i) Spatial Distance.

(a) The distance moved by O_S in his own frame of reference is zero.

(b) The distance moved by O_T, as assessed by O_S, using a unit measuring rod, (all measurements taken with the rod stationary), is *D*.

(c) The distance moved by O_T in his own frame of reference is zero.

(d) The apparent distance moved by O_S, as assessed by O_T using a unit measuring rod, (all measurements taken with the rod moving with a velocity in Pseudo-Euclidean space of *v*), is $D/\left(1-v^2/c^2\right)^{1/2}$ by virtue of the Lorentz-Fitzgerald contraction of the measuring rod.

(b) and (d) show that the spatial dimensions of the two reference frames are not symmetric.

(ii) Temporal Distance.

(a) The temporal distance travelled by O_S in his own frame of reference is $c\Delta t$.

(b) The temporal distance travelled by O_T, as measured by O_S is given by (2.9), i.e. $c\left(1-v^2/c^2\right)^{1/2}\Delta t$.

(c) The temporal distance travelled by O_S as measured by O_T is given by (2.12) and is, with (2.8) and (2.10) incorporated, $c\left(1-v^2/c^2\right)^{1/2}\Delta t$.

(d) The temporal distance moved by O_T as measured in his own frame of reference is $c'\Delta t' = c\left(1-v^2/c^2\right)\Delta t$.

These results show that the temporal dimensions of the two reference frames are not symmetric. This non-symmetry of distance between the two reference frames is the result of the rotation of the Existence Velocity vector of O_T.

(iii) Time.

The passage of time is not a symmetric parameter between the two reference frames as exemplified by the time dilatation effect, and confirmed by the results of this analysis, albeit each twin can correctly identify the age difference.

(iv) Velocity.

Section 2.1 shows that spatial velocities in the two reference frames are symmetric, (in this case diametrically symmetric, same magnitude, different directions). Hence the velocity of light is the same in both frames. This symmetry arises because the non-symmetry of spatial distance is offset by the non-symmetry in time.

The temporal velocities of O_S and O_T are c and $c\left(1-v^2/c^2\right)^{1/2}$ respectively and therefore not symmetric. This non-symmetry exists because the non-symmetries in temporal distance and time are concurrent as shown by (ii)(d) above.

4.0 Conclusions.

The use of the Relativistic Domain mathematical formulation of the Special Theory, has enabled this paper to easily show that the "Twin Paradox" is in fact not a paradoxical problem at all, as each twin can correctly assess the age difference as a result of the journey of one of them. Albeit this has been achieved by allowing each observer to assess the temporal velocity of the other, a measurement not practicable in real world problems, and only permitted here because of the nature of the experiment.
The age difference correctly identified by each twin is essentially the result of the non-symmetry of temporal velocity, while importantly, the velocity of light is the same in both reference frames because of the symmetry of spatial velocity between them.

Also, it is noted that the analysis here does not require the necessity of considering the acceleration involved in the motion, nor the effect of gravity.

Finally, it has also been shown that the nature of symmetry between the two reference frames is not just a simple overall one, but depends upon the dimensions contained therein and the parameter involved.

References.

[1] NPA, *Twin Paradox Report*, Online.

[2] Wikipedia, *Twin Paradox*, Online.

[3] P.G.Bass, *The Special Theory of Relativity - A Classical Approach,* www.relativitydomains.com.