THE VARIABILITY OF TEMPORAL RATE IN PSEUDO-EUCLIDEAN SPACE-TIME

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Abstract

This short paper, via analysis of the characteristics of the spectral Doppler shift of a plain wavefront, investigates the purported variability of temporal rate in a moving reference frame in Pseudo-Euclidean Space-Time.

1 Introduction.

A central feature of Albert Einstein's Special Theory of Relativity is the variability of temporal rate with spatial velocity. The degree of variability is a second order relativistic one and leads directly to a similar level of variability of all other physical parameters that exhibit either direct or indirect dependence on time.

There have been many attempts to measure this level of variability, of temporal rate itself, as reported in [1], and of other parameters as reported in [2] and [3]. These references report successful experiments, but there have been others, as reported in [4], in which the experiments have resulted in a negative conclusion. The difficulty lies in the fact that all of these experiments have been trying to measure a second order relativistic effect of an event exhibiting a relatively slow spatial velocity, compared to the velocity of light, while excluding, and/or allowing for, other extraneous effects.

Contrarily, there have also been attempts to show that the characteristics of Pseudo-Euclidean Space-Time, (designated D_0 in this series of papers), can be adequately described without the necessity for temporal rate variability. Reference [4] is one such.

To resolve the issue, what is needed is a way of determining the veracity of a representative Pseudo-Euclidean reference frame, with or without temporal rate variability, via comparison of its characteristics with empirical measurements in which only first order relativistic events are present. This paper presents such a determination by the analysis of the spectral Doppler shift of a plain wavefront.

In the interests of brevity, unless necessary for complete clarity, a parameter will only be defined in this paper if it has not already been so in [5] or [6] with which familiarity is assumed.

2 The Characteristics of Spectral Doppler Shift in Pseudo-Euclidean Space-Time.

The investigation will be conducted by deriving a general expression for the spectral Doppler shift of a plain wave in moving axes, which can then be applied to various reference frames of interest, to determine their veracity as representative of Pseudo-Euclidean Space Time.

The equation of a spherical wave emanating from a stationary source in a stationary linear reference frame in D_0 is,

$$x_1^2 + x_2^2 + x_3^2 - c^2t^2 = 0 \tag{2.1}$$

In another linear reference frame moving with constant linear velocity v, the equation of the same wave is,

$$x_1'^2 + x_2'^2 + x_3'^2 - c'^2t'^2 = 0 \tag{2.2}$$

To ensure that all possible moving frames of reference are considered, (2.2) assumes that space, temporal rate and the measured velocity of propagation of the wave can all be variables.

By the Principle of Relativity, (2.1) and (2.2) can be equated thus,

$$x_1^2 + x_2^2 + x_3^2 - c^2t^2 = x_1'^2 + x_2'^2 + x_3'^2 - c'^2t'^2 = 0 \tag{2.3}$$

For the respective sides of (2.3),

$$\lambda f = c \qquad \textit{and} \qquad \lambda' f' = c' \tag{2.4}$$

$$f = \frac{1}{\vartheta} = \frac{n}{t} \qquad \textit{and} \qquad f' = \frac{1}{\vartheta'} = \frac{n'}{t'} \tag{2.5}$$

where

ϑ and ϑ' are wave cycle intervals.

n and n' are the number of cycles in a time t and t' respectively.

From (2.4) and (2.5)

$$c^2t^2 = n^2\lambda^2 \qquad \textit{and} \qquad c'^2t'^2 = n'^2\lambda'^2 \tag{2.6}$$

Inserting (2.6) into (2.3) gives

$$x_1^2 + x_2^2 + x_3^2 - n^2\lambda^2 = x_1'^2 + x_2'^2 + x_3'^2 - n'^2\lambda'^2 = 0 \tag{2.7}$$

This is the same as [4] Eq.(7) and shows that the same expression of wave propagation applies in all the various types of reference frame to be considered. In particular, the specific temporal rate invariant one in [4], and, amongst others, the temporal rate variant one here.
To simplify the analysis, assume now that the wave is a plain one moving in the x_1 direction, and that the moving reference frame is moving away from the source along the same axis at a constant linear velocity v. Consequently, any Doppler shift should be towards the red end of the spectrum. Eq(2.7) then reduces to, (dropping the subscripts),

$$x^2 - n^2\lambda^2 = x'^2 - n'^2\lambda'^2 = 0 \tag{2.8}$$

Eq(2.8) can be re-arranged to give,

$$\lambda'^2 = \lambda^2 \left(\frac{n^2}{n'^2} + \frac{x'^2 - x^2}{n'^2\lambda^2} \right) \tag{2.9}$$

From (2.6)

$$n'^2\lambda^2 = \frac{n'^2}{n^2}c^2t^2 \tag{2.10}$$

which when inserted into (2.9) gives

$$\lambda'^2 = \lambda^2 \frac{n^2}{n'^2} \left(1 + \frac{x'^2 - x^2}{c^2t^2} \right) \tag{2.11}$$

which is an expression for the Doppler shift extant in any linear moving reference frame of interest. First consider the simplest.

2.1 In a Galilean-Newtonian Frame of Reference.

In this frame both space and temporal rate are invariant. Therefore, because of the temporal rate invariance,

$$n = n' \tag{2.12}$$

and because of the invariance of space,

$$x' = x - vt \tag{2.13}$$

Substitution of (2.12) and (2.13) into (2.11) yields

$$\lambda'^2 = \lambda^2 \left(1 - \frac{2xv}{c^2 t} + \frac{v^2}{c^2}\right) \tag{2.14}$$

and when $x = ct$, the distance moved by the wavefront in a time t in the stationary reference frame,

$$\lambda' = \lambda \left(1 - \frac{v}{c}\right) \tag{2.15}$$

This is a shift to the violet end of the spectrum and is contrary to empirical results. Also from (2.13),

$$\frac{dx'}{dt} = \frac{dx}{dt} - v \tag{2.16}$$

so that when $dx/dt = c$,

$$\frac{dx'}{dt} = c\left(1 - \frac{v}{c}\right) \tag{2.17}$$

The velocity of wave propagation in the moving frame is therefore different from that in the stationary frame. This is contrary to the Michelson - Morley empirical result. The above two anomalies eliminated the Galilean - Newtonian frame of reference as representative of Pseudo - Euclidean Space - Time.

2.2 In a Lorentz - Fitzgerald Frame of Reference.

In this frame, temporal rate is invariant, but space is allowed to vary such as to ensure that wave propagation velocity is the same in both the stationary and the moving frames of reference. Thus (2.12) applies and

$$x' = \frac{x - vt}{\left(1 - \frac{v}{c}\right)} \tag{2.18}$$

Clearly when $dx/dt = c$, (2.18) gives

$$\frac{dx'}{dt} = c \tag{2.19}$$

Substitution of (2.12) and (2.18) into (2.11) gives,

$$\lambda'^2 = \lambda^2 \left\{ 1 + \frac{\frac{(x-vt)^2}{\left(1-\frac{v}{c}\right)^2} - x^2}{c^2 t^2} \right\} \tag{2.20}$$

which reduces to,

$$\lambda' = \lambda \tag{2.21}$$

Thus there is no Doppler shift in this reference frame. This is because the relativistic variation of space exactly counter-balances the perceived variation in the wavelength of the wave. This result is contrary to empirical results and eliminates this reference frame as representative of Pseudo-Euclidean Space-Time.

2.3 A Temporal Rate Invariant Reference Frame Incorporating Doppler Shift.

In this frame temporal rate is again invariant, but Doppler shift, conformant with empirical results, is incorporated. From this the characteristics of the moving reference frame can be derived. Thus (2.12) applies and,

$$\lambda' = \lambda\left(1+\frac{v}{c}\right) \tag{2.22}$$

Eq(2.22) is taken from [4] Eq(2). Thus from (2.11), (2.12) and (2.22)

$$1+\frac{x'^2-x^2}{c^2t^2} = \left(1+\frac{v}{c}\right)^2 \tag{2.23}$$

Solving for x' gives,

$$x'^2 = x^2 + v\,(2c+v)\,t^2 \tag{2.24}$$

Eq(2.24) concurs with [4]Eq(10). Differentiating (2.24) with respect to the time gives,

$$\frac{dx'}{dt} = \frac{x\frac{dx}{dt} + v\ (2c+v)\,t}{\{\,x^2 + v\ (2c+v)\,t^2\}^{1/2}} \tag{2.25}$$

and when $x = ct$ and therefore $dx/dt = c$, (2.25) reduces to,

$$\frac{dx'}{dt} = c\left(1+\frac{v}{c}\right) \tag{2.26}$$

Thus the velocity of the wave in the moving frame is no longer equal to that in the stationary frame. This is contrary to the Michelson - Morley experimental result and consequently this reference frame cannot be representative of Pseudo-Euclidean Space-Time.

2.4 In an Einstein - Minkowski Reference Frame.

In this reference frame both space and temporal rate are purported to be variable. The relationship with the stationary frame is given by the Lorentz transformations, viz,

$$x' = \frac{x-vt}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \tag{2.27}$$

and

$$t' = \frac{t-x\frac{v}{c^2}}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \tag{2.28}$$

and when $dx/dt = c$, it is well known and easily shown that,

$$\frac{dx'}{dt'} = c \tag{2.29}$$

It is not possible to derive the Doppler shift in this reference frame from (2.11) et al, and therefore the same procedure as in the last Section will be adopted. The Doppler shift purported in this frame is given by,

$$\lambda' = \lambda\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{1/2} \tag{2.30}$$

as derived from [4]Eq(3). Insertion of (2.30) into (2.11) gives,

$$\frac{n^2}{n'^2}\left(1+\frac{x'^2-x^2}{c^2t^2}\right) = \frac{1+\frac{v}{c}}{1-\frac{v}{c}} \tag{2.31}$$

From (2.6)

$$\frac{n^2}{n'^2} = \frac{\lambda'^2 t^2}{\lambda^2 t'^2} \tag{2.32}$$

Inserting (2.28) and (2.30) into (2.32) gives,

$$\frac{n^2}{n'^2} = \frac{\left(1+\frac{v}{c}\right)}{\left(1-\frac{v}{c}\right)} \frac{\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{xv}{tc^2}\right)^2} \tag{2.33}$$

Substituting (2.33) into (2.31) gives,

$$1 + \frac{x'^2 - x^2}{c^2 t^2} = \frac{\left(1-\frac{xv}{tc^2}\right)^2}{\left(1-\frac{v^2}{c^2}\right)} \tag{2.34}$$

which reduces to,

$$x' = \frac{x - vt}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \tag{2.35}$$

which concurs with (2.27).
This effectively shows that this reference frame, in which the velocity of the wave is the same as in the stationary reference frame, also produces the correct value for Doppler shift.

3 Conclusions.

The two parameters that have been used as measures of veracity, are spectral Doppler shift and the velocity of light. Both of these are capable of measurement to an extremely high accuracy, certainly high enough to discern first order relativistic variation. Consequently, the primary conclusion to be drawn from the brief investigation conducted here, is that, because it is incapable of producing the above physical events to even a first degree of relativistic accuracy, a temporal rate invariant reference frame does not possess the appropriate characteristics to represent Pseudo-Euclidean Space-Time.

The reference frame incorporating temporal rate variability, the Einstein-Minkowski reference frame, does possess satisfactory attributes in that it firstly exhibits the velocity of light as an invariant under all conditions, and secondly predicts the gross Doppler effect as an approximation when the velocity of the moving reference frame is small. The second order relativistic effect only becomes apparent when the velocity of the moving frame is a significant fraction of the velocity of light. The consequence of course in such a reference frame, is that all other parameters that possess dependence on time, will also exhibit similar second order relativistic variability. The one most referred to of course is mass. An example of a number of others is the subject of [5].

This paper cannot be said to "prove" that the Einstein-Minkowski reference frame is the only one suitable to represent Pseudo-Euclidean Space Time, but it is evident that it does show that a frame incorporating temporal rate invariance cannot. The "proof" that a temporally variant frame is fully viable, will only be approached when a number of parameters theoretically possessing second order relativistic variability are successfully confirmed via undisputed experimental measurement. The most likely candidates are those used here, time itself, and spectral Doppler shift. Both of these could be the subject of precise measurement from highly stable sources on a long term space exploration project. The results of these measurements would be transmitted to Earth, and compared with those from identical sources maintained under similarly stable conditions. The second order variations of temporal rate would be manifested as an increasing divergence of measured elapsed time, the rate of which would depend upon the velocity of the craft. The second order Doppler shift would become apparent as the velocity of the craft increased to a suitable fraction of that of light.

Finally, it has been stated, [4], that Einstein's incorporation of temporal rate variability was by way of an assumption towards ensuring that the velocity of light was absolutely invariant in both stationary and moving reference frames. It is to be noted that in [6], as a result of the new mathematical representation of Minkowski space-time, temporal rate variability occurs as a natural consequence of the primary criterion of existence within that space-time, and thereby leads to a simpler derivation of all aspects of the Special Theory.

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