The Relativistic Characteristics of a Spinning Spherical Mass in Pseudo-Euclidean Space-Time, D_0

Peter G. Bass

Abstract

This paper investigates some characteristics of spherical masses when spinning at high angular rates within the Relativistic Space-Time Domain D_0, and discusses some of the potential implications.

1 Introduction.

The rectilinear motion of a body of matter is the only motion in which the effects of relativistic variation affect the whole of the mass identically and simultaneously. This is because the whole of the mass experiences exactly the same velocity magnitude at the same time. When the motion is along a curved path, different parts of the mass experience different velocity magnitudes. Therefore the relativistic effects are graduated throughout the body, along the radius vector from the centre of curvature to the path of the motion. When the radius of curvature of the path is much greater than the physical dimensions of the body, some relativistic effects can be treated as in the rectilinear case in an acceptable approximation. This scenario was briefly analysed in [1]. Clearly the greatest relativistic gradient effect occurs when the above radius vector magnitude is zero, and the mass is simply spinning.

This scenario is investigated in this paper for a spherical homogeneous mass. Parameters analysed for relativistic variability are mass, moment of inertia and associated radius of gyration, angular momentum, spin energy, volume, surface area, average matter density and circumference on the spin plane. While most of the effects are of mathematical interest only, there is one relativistic variation which may have significant implications in both the microscopic and macroscopic realms of existence. This is discussed at some length.
The effect of a combination of a rectilinear translational velocity and high angular rate spin on the relativistic mass is also analysed.
Also demonstrated in this paper is a new mathematical representation of centripetal acceleration.

It is assumed for the purpose of mathematical demonstration of the said effects, that the mechanical integrity of the spinning mass is not violated.

In the interests of brevity, unless necessary for complete clarity, a parameter will only be defined in this paper, if it has not already been so in [1] or [2] with which familiarity is assumed.

2 The Relativistic Characteristics of Rapidly Spinning Spherical Bodies.

2.1 Mass

To determine the relativistic effects upon the mass of a spherical body spinning at a rapid angular rate, consider Fig. 2.1 below.

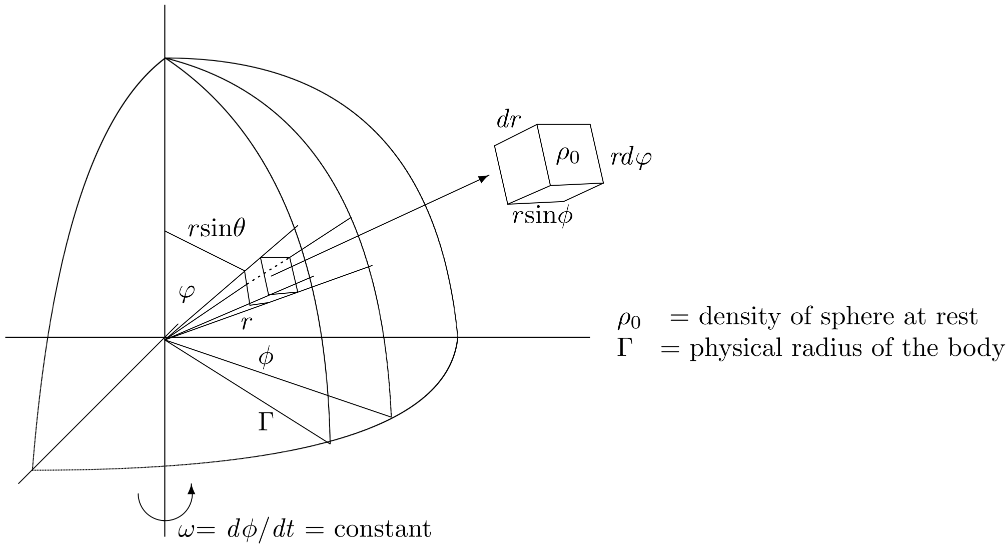

Fig. 2.1 Section of a Spinning Spherical Body

In Fig. 2.1 the volume of the elemental is

$$W_{ele} = r^2 \sin\varphi \, d\phi \, d\varphi \, dr \tag{2.1}$$

The rest mass of the elemental is then

$$m_{0(ele)} = \rho_0 r^2 \sin\varphi \, d\phi \, d\varphi \, dr \tag{2.2}$$

The velocity of the elemental due to the spin of the body is

$$v_{ele} = \omega r \sin\varphi \tag{2.3}$$

The energy mass of the elemental is therefore

$$m_{ele} = \frac{m_{0(ele)}}{\left(1 - \dfrac{v_{ele}^2}{c^2}\right)^{1/2}} = \frac{\rho_0 r^2 \sin\varphi \, d\phi \, d\varphi \, dr}{\left(1 - \dfrac{\omega^2 r^2 \sin^2\varphi}{c^2}\right)^{1/2}} \tag{2.4}$$

The toroidal energy mass is obtained by integrating (2.4) with respect to ϕ over the limits $0 \leq \phi \leq 2\pi$ to give

$$m_{tor} = \frac{2\pi\rho_0 r^2 \sin\varphi \, d\varphi \, dr}{\left(1 - \dfrac{\omega^2 r^2 \sin^2\varphi}{c^2}\right)^{1/2}} \tag{2.5}$$

To complete the exercise of obtaining the energy mass of the spinning body it is first necessary to integrate (2.5) with respect to φ between the limits $0 \leq \varphi \leq \pi$. Then it is necessary to integrate the result with respect to r over the limits $0 \leq r \leq \Gamma$. While these integrations can be performed in an exact and rigorous manner it is a somewhat lengthy and involved process. It is simpler to make a relativistic approximation at this point which is then "reversed" after the integrations are completed. This process is valid because the expansions are absolutely convergent series and the result is accurate to about 1.12% as $\omega\Gamma \Rightarrow c$. The approximation is therefore quite adequate to illustrate the significance of the result.

Thus binomially expanding the square root in (2.5), the first integral specified above becomes

$$m_{sh} = 4\pi\rho_0 r^2 dr \int_0^{\pi/2} \left(\sin\varphi + \frac{\omega^2 r^2}{2c^2}\sin^3\varphi - \frac{3}{8}\frac{\omega^4 r^4}{c^4}\sin^5\varphi + ...\right) d\varphi \tag{2.6}$$

Eq.(2.6) integrates to

$$m_{sh} = 4\pi\rho_0 r^2 \left(1 + \frac{\omega^2 r^2}{3c^2} - \frac{\omega^4 r^4}{5c^4} + ...\right) dr \tag{2.7}$$

Integrating (2.7) with respect to r as specified then yields

$$m_{sph} = \frac{4}{3}\pi\rho_0\Gamma^3 \left(1 + \frac{\omega^2\Gamma^2}{5c^2} - \frac{3\omega^4\Gamma^4}{35c^4} + ...\right) \tag{2.8}$$

for the energy mass of the spinning sphere.
Performing the reverse expansion on (2.8), using just the first relativistic term gives

$$m_{sph} \cong \frac{m_0}{\left(1 - \frac{2}{5}\frac{\omega^2\Gamma^2}{c^2}\right)^{1/2}} \tag{2.9}$$

where the classical expression for the rest mass has also been inserted. Eq.(2.9) can be written

$$m_{sph} \cong \frac{m_0}{\left(1 - \frac{\omega^2\Gamma_{gyr}^2}{c^2}\right)^{1/2}} \tag{2.10}$$

where

$\Gamma_{gyr}$is the classical radius of gyration of a stationary spherical homogeneous mass, $(=\sqrt{\frac{2}{5}}\Gamma)$.

Eq.(2.10) is the energy mass of the spinning body wherein the relativistic mass increase is the stored kinetic energy induced by the applied accelerating torque.
The important feature about this result is apparent in (2.9) where it can be seen that if $\omega\Gamma = c$ then the mass becomes

$$m_{sph} \cong \sqrt{\frac{5}{3}} m_0 \tag{2.11}$$

and is a maximum.

Thus the surface of the sphere at the spin circumference, can be accelerated to the terminal velocity of D_0, ($\sim$ the speed of light), while the energy mass of the sphere remains finite. This is solely due to the distributed nature of the mass and that each toroidal element between the centre and the spin circumference is spinning at a velocity lower than $\omega\Gamma$. Because the energy mass of the sphere is finite under this condition, only a finite amount of energy has been applied to reach this state, and it would therefore be possible to apply additional energy in the form of an accelerating torque, to further increase the spin rate. The consequence of this would be that the surface of the sphere at the spin circumference would tend to exceed the terminal velocity of D_0, and thus contravene its primary criterion of existence, i.e. that the maximum spatial velocity attainable in that Domain is the velocity constant c. To avoid this the mass must therefore lose energy. In a future paper concerning the existence of de Broglie matter waves in D_0, it will be shown that matter can only exist at the terminal velocity of D_0 as pure kinetic energy, possibly in the form of photons/electromagnetic radiation. It is therefore proposed that a spherical mass induced to spin such that the surface at the spin circumference tends to exceed the terminal velocity in D_0, avoids that anomaly by losing kinetic energy at that surface, by converting it to radiant energy at some frequency and wavelength proportional to the energy applied to increase the angular rate. The emission of the spectra would be in accordance with the quantum laws of Planck and de Broglie.

This phenomenon may have significant implications in Cosmology and Atomic Structure Theory, and is discussed further along those lines in the concluding remarks.

2.2 Moment of Inertia and Radius of Gyration.

Both of these parameters will, due to the spinning motion, be subject to relativistic variation. First the moment of inertia.
The moment of inertia of the elemental mass of (2.4) is

$$I_{ele} = \frac{\rho_0 r^4 \sin^3\varphi \, d\phi \, d\varphi \, dr}{\left(1 - \dfrac{\omega^2 r^2 \sin^2\varphi}{c^2}\right)^{1/2}} \tag{2.12}$$

Proceeding as in the last Section the moment of inertia of the toroid is determined by integration of (2.12) with respect to ϕ between the limits $0 \leq \phi \leq 2\pi$ to give

$$I_{tor} = \frac{2\pi\rho_0 r^4 \sin^3\varphi \, d\varphi \, dr}{\left(1 - \dfrac{\omega^2 r^2 \sin^2\varphi}{c^2}\right)^{1/2}} \tag{2.13}$$

Again proceeding as in the previous Section and binomially expanding the square root in (2.13) gives

$$I_{tor} = 2\pi\rho_0 r^4 \sin^3\varphi \left(1 + \frac{\omega^2 r^2 \sin^2\varphi}{2c^2} + \frac{3}{8}\frac{\omega^4 r^4 \sin^4\varphi}{c^4} + ...\right) d\varphi \, dr \tag{2.14}$$

The moment of inertia of the spherical shell then becomes

$$I_{sh} = 4\pi\rho_0 r^4 dr \int_0^{\pi/2} \left(\sin^3\varphi + \frac{\omega^2 r^2 \sin^5\varphi}{2c^2} + \frac{3}{8}\frac{\omega^4 r^4 \sin^7\varphi}{c^4} + ...\right) d\varphi \tag{2.15}$$

These are all standard integrals which yield

$$I_{sh} = \frac{8}{3}\pi\rho_0 r^4 \left(1 + \frac{2}{5}\frac{\omega^2 r^2}{c^2} + \frac{9}{35}\frac{\omega^4 r^4}{c^4} + ...\right) dr \tag{2.16}$$

Now, integrating (2.16) with respect to r between the limits $0 \leq r \leq \Gamma$ gives the moment of inertia of the spinning sphere as

$$I_{sph} = \left(\frac{4}{3}\pi\rho_0\Gamma^3\right)\left(\frac{2}{5}\Gamma^2\right)\left(1 + \frac{2}{7}\frac{\omega^2\Gamma^2}{c^2} + \frac{1}{7}\frac{\omega^4\Gamma^4}{c^4} + ...\right) \tag{2.17}$$

Reversing the earlier expansion process, using just he first relativistic term, finally gives

$$I_{sph} \cong \frac{I_0}{\left(1 - \frac{4}{7}\frac{\omega^2\Gamma^2}{c^2}\right)^{1/2}} \tag{2.18}$$

So that when $\omega\Gamma = c$, (2.18) becomes

$$I_{sph} \cong \sqrt{\frac{7}{3}} I_0 \tag{2.19}$$

and is a maximum.
To determine the relativistically corrected radius of gyration, note that according to classical mechanics the relativistically corrected moment of inertia would be given by

$$I_{sph} = m_{sph}\left(\Gamma^*_{gyr}\right)^2 \tag{2.20}$$

Where

Γ^*_{gyr} is the relativistically corrected radius of gyration.

Inserting (2.9) for the energy mass, equating the result to (2.18) and solving for the relativistic radius of gyration yields

$$\left(\Gamma^*_{gyr}\right)^2 \cong \frac{I_0\left(1-\frac{2}{5}\frac{\omega^2\Gamma^2}{c^2}\right)^{1/2}}{m_0\left(1-\frac{4}{7}\frac{\omega^2\Gamma^2}{c^2}\right)^{1/2}} \tag{2.21}$$

By taking only second order relativistic terms, (2.21) reduces to

$$\Gamma^*_{gyr} \cong \frac{\Gamma_{gyr}}{\left(1-\frac{3}{35}\frac{\omega^2\Gamma^2}{c^2}\right)^{1/2}} \cong \frac{\Gamma_{gyr}}{\left(1-\frac{3}{14}\frac{\omega^2\Gamma^2_{gyr}}{c^2}\right)^{1/2}} \tag{2.22}$$

When $\omega\Gamma = c$, (2.22) becomes

$$\Gamma^*_{gyr} \cong \sqrt{\frac{35}{32}}\Gamma_{gyr} \tag{2.23}$$

and is a maximum.

Thus both the moment of inertia and the radius of gyration are relativistically variable parameters. The reason is because as the spin rate of the spherical body increases, the outer part of the body, which contains the greater part of the mass, also experiences a higher spin velocity so absorbing the greater proportion of kinetic energy. This results in that part of the body gaining the greater part of the relativistic mass increase, so causing an increase in the radius of gyration and thereby the moment of inertia.

2.3 Angular Momentum.

From (2.18) the relativistic angular momentum of the spinning mass is given by

$$T_{sph} = \omega I_{sph} \cong \frac{T_0}{\left(1-\frac{4}{7}\frac{\omega^2\Gamma^2}{c^2}\right)^{1/2}} \tag{2.24}$$

and when $\omega\Gamma = c$, this reduces to

$$T_{sph} \cong \sqrt{\frac{7}{3}}T_0 \tag{2.25}$$

Eq. (2.25) is the maximum angular momentum that can be obtained by any spinning homogeneous spherical mass within Pseudo-Euclidean Space-Time, (D_0).

2.4 Spin Energy.

The kinetic energy gained from the accelerating torque by the spinning mass is given by the classical equation

$$E_k = \frac{\omega^2 I_{sph}}{2} \tag{2.26}$$

and from (2.18) this becomes

$$E_k \cong \frac{\omega^2 I_0}{2\left(1-\frac{4}{7}\frac{\omega^2\Gamma^2}{c^2}\right)^{1/2}} \tag{2.27}$$

The maximum is again given when $\omega\Gamma = c$, viz.

$$E_k \cong \sqrt{\frac{7}{75}}m_0c^2 \tag{2.28}$$

This is a little over 30% of the rest energy, quite a low value for a relativistic energy maximum. The application of further accelerating torque does not add kinetic energy to (2.28) but, as proposed in Section 2.1, is radiated away by that part of the surface experiencing the terminal velocity.

2.5 Volume, Surface Area and Average Matter Density.

Using the results of the previous Sections, it is easy to show that the volume of a spinning sphere, measured in units associated with stationary axes in D_0 is

$$W_{sph} \cong \frac{W_0}{\left(1+\frac{2}{5}\frac{\omega^2\Gamma^2}{c^2}\right)^{1/2}} \cong \frac{W_0}{\left(1+\frac{\omega^2\Gamma^2_{gyr}}{c^2}\right)^{1/2}} \tag{2.29}$$

and the surface area

$$\Lambda_{sph} \cong \frac{\Lambda_0}{\left(1+\frac{2}{3}\frac{\omega^2\Gamma^2}{c^2}\right)^{1/2}} \tag{2.30}$$

It is interesting to note that these last two parameters, volume and surface area reduce as spin rate increases in contrast to the other parameters studied here which increase. This is caused by the Lorentz/Fitzgerald contraction of the circumference as is shown in Section 2.6 below.
From (2.9) and (2.29) it is clear that the average density of a spinning sphere is also a relativistic variable given by

$$\rho \cong \rho_0 \frac{\left(1+\frac{2}{5}\frac{\omega^2\Gamma^2}{c^2}\right)^{1/2}}{\left(1-\frac{2}{5}\frac{\omega^2\Gamma^2}{c^2}\right)^{1/2}} \cong \frac{\rho_0}{\left(1-\frac{2}{5}\frac{\omega^2\Gamma^2}{c^2}\right)} \cong \frac{\rho_0}{\left(1-\frac{\omega^2\Gamma^2_{gyr}}{c^2}\right)} \tag{2.31}$$

and when $\omega\Gamma = c$

$$\rho_{\max} \cong \frac{5}{3}\rho_0 \tag{2.32}$$

Eq.(2.32) is the maximum average density attainable by a spinning spherical homogeneous mass.

2.6 The Circumference in the Spin Plane.

Consider the elemental in Fig. 2.2 below

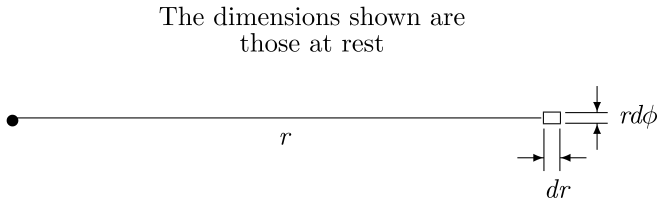

Fig. 2.2 Rotating Elemental.

In Fig. 2.2 $rd\phi$ is the width of the elemental at rest. The effect of Lorentz/Fitzgerald contraction is to reduce dimensions in the direction of motion. Thus the width of the above elemental rotating at a constant angular rate of ω would, in stationary axes in D_0, become

$$w = rd\phi\left(1-\frac{\omega^2 r^2}{c^2}\right)^{1/2} \tag{2.33}$$

If now the elemental in Fig. 2.2 is extended to become a rotating toroid, i.e. integrating $d\phi$ over the limits $0 \le \phi \le 2\pi$, then (2.33) becomes

$$w_{cir} = 2\pi\, r\left(1-\frac{\omega^2 r^2}{c^2}\right)^{1/2} \tag{2.34}$$

An alternative way of representing this effect is as follows.
The axes attached to a body in motion are sometimes referred to as having rotated into the temporal dimension, [1], [3]. To depict the effect proposed here, if the spin circumference of the rotating toroid is represented using Fig. 2.3 below

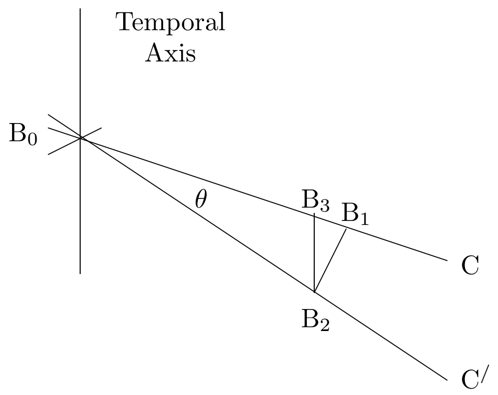

Fig. 2.3 Representation of Circumference in Relativistic Axes

In Fig. 2.3

C is an axis stationary in D_0.

$C^/$ is an axis moving with the velocity of the toroidal circumference, i.e. ωr. This axis is therefore effectively attached to the rotating toroid.

θ is the spatial axis rotation angle into the temporal dimension and where $sin\theta = \omega r/c$

If the circumference of the toroid is laid out on the two above axes, then the distance $0 \rightarrow B_1$ on the C axis is the circumference of the toroid at rest. i.e. $2\pi r$. The distance $0 \rightarrow B_2$ is the circumference of the toroid on the moving axes measured in the units of the moving axis. This therefore also measures the circumference as at rest. Consequently,

$$|\,0 \rightarrow B_1| = |\,0 \rightarrow B_2| = 2\pi r \tag{2.35}$$

The distance $0 \rightarrow B_3$ is the circumference of the spinning toroid measured in the stationary C axis in D_0. Thus

$$|\,0 \rightarrow B_3| = |\,0 \rightarrow B_2| \cos\theta \tag{2.36}$$

and by virtue of (2.35)

$$\begin{aligned} |\,0 \rightarrow B_3| &= |\,0 \rightarrow B_1| \cos\theta \\ &= |\,0 \rightarrow B_1| \left(1 - \frac{\omega^2 r^2}{c^2}\right)^{1/2} \\ &= 2\pi r \left(1 - \frac{\omega^2 r^2}{c^2}\right)^{1/2} \end{aligned} \tag{2.37}$$

which is identical to (2.34).

This result leads to the apparent anomaly that in the limit, when the outer surface of the toroid reaches the terminal velocity of D_0, i.e. $\omega r = c$, the circumference of this outer surface becomes zero for a finite radius. In the axis attached to the spinning toroid the circumference remains unchanged at $2\pi r$. This is however, no different from the reduction to zero of length in a linear Lorentz/Fitzgerald contraction.

2.7 Time Dilatation and its Radial Gradient

From relativity theory it is well known that a mass in motion at some velocity v experiences a slowing down in the rate of passage of time given by

$$\frac{dt^{/}}{dt} = \left(1 - \frac{v^2}{c^2}\right)^{1/2} \tag{2.38}$$

where

t$^{/}$ is the time measured by the body in motion.

If the motion concerned is the spinning of the mass then (2.38) becomes

$$\frac{dt^{/}}{dt} = \left(1 - \frac{\omega^2 r^2}{c^2}\right)^{1/2} \tag{2.39}$$

The rate of passage of time as measured on the rotating mass is now a relativistic function of the radial distance r and so possesses a radial gradient in the spin plane. Putting

$$\frac{dt^{/}}{dt} = u \tag{2.40}$$

then from (2.39) and (2.40)

$$\frac{du}{dr} = -\frac{\omega^2 r}{uc^2} \tag{2.41}$$

so that

$$c^2 u \frac{du}{dr} = -\omega^2 r \tag{2.42}$$

This is similar to the gravitational case where the term on the left is shown in [2] Eq.(3.17) and [2] Eq. (4.4) et seq. to represent the gravitational Acceleration Potential. In the case shown here it represents the centripetal acceleration of a spinning body in D_0, which is therefore seen to be the result of the radial temporal dilatation gradient due to the spin motion. The similarity is however, only a mathematical one because the Acceleration Potential of D_1, i.e. gravity, is the result of the time dilatation gradient produced by the gravitational source, i.e. there is no external force applied to produce this effect. Centripetal acceleration on the other hand is caused by the change in direction of the rotating body as it spins about the centre of rotation. An external force is exerted on it by whatever means it is connected to the centre of rotation. Eq. (2.42) is consequently only an alternative mathematical expression for centripetal acceleration.

2.8 Combined Translational and Spin Motions - Relativistic Mass.

When a spinning spherical mass also possesses significant translational motion, the situation becomes a little more complex. Only the mass of the body will be analysed under this condition. Other parameters will exhibit similar characteristics.
Consider a plan view of Fig. 2.1

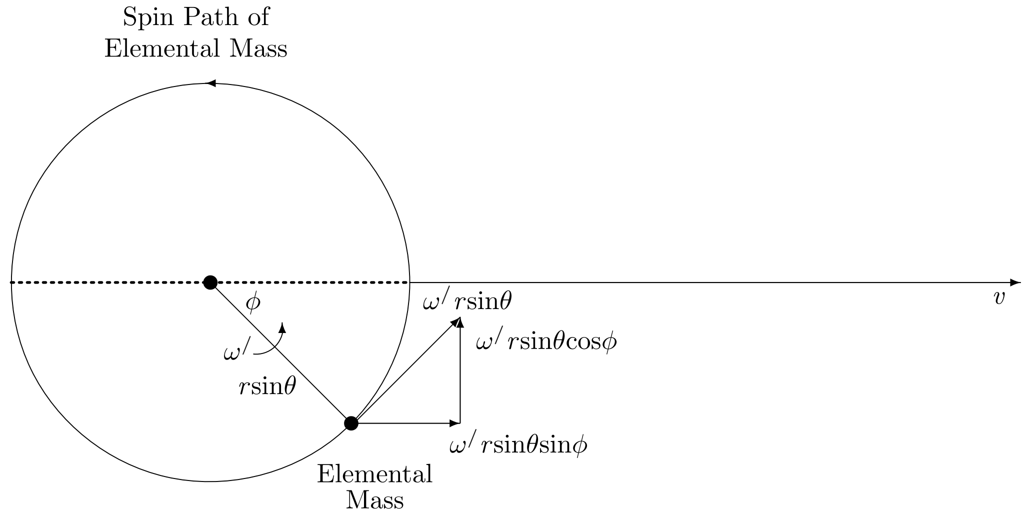

Fig. 2.4 Spinning Elemental Mass Plus Translational Velocity

From Fig. 2.4 the spin velocity of the elemental mass, normal to the direction of the translational motion is

$$v_n = \omega' r \sin\varphi \cos\phi \tag{2.43}$$

where

$\omega^{/}$ is now the spin rate in axes moving translationally with the body.

The spin velocity in the same direction as the translational velocity is

$$v_s = \omega' r \sin\varphi \sin\phi \tag{2.44}$$

The total translational velocity of the elemental is then the relativistic sum of (2.44) and v, i.e.

$$v^* = \frac{v + v_s}{1 + \frac{v v_s}{c^2}} \tag{2.45}$$

The magnitude of the total velocity of the elemental is then

$$v_{ele} = \left(v_n^2 + v^{*\,2}\right)^{1/2} = \left\{ \left(\omega' r \sin\varphi \cos\phi\right)^2 + \frac{\left(v + \omega' r \sin\varphi \sin\phi\right)^2}{\left(1 + \frac{v\omega' r \sin\varphi \sin\phi}{c^2}\right)^2} \right\}^{1/2} \tag{2.46}$$

The mass of the elemental at rest is given by (2.2) so that the energy mass of the elemental in this scenario is

$$m_{ele} = \frac{m_0}{\left(1 - \frac{v_{ele}^2}{c^2}\right)^{1/2}} = \frac{\rho_0 r^2 \sin\varphi \, d\phi \, d\varphi \, dr}{\left\{ 1 - \frac{\left(\omega' r \sin\varphi \cos\phi\right)^2 + \frac{\left(v + \omega' \sin\varphi \sin\phi\right)^2}{\left(1 + \frac{v\omega' r \sin\varphi \sin\phi}{c^2}\right)^2}}{c^2} \right\}^{1/2}} \tag{2.47}$$

First, take a relativistic approximation of the second term of the denominator retaining only second order terms. This gives simply

$$m_{ele} = \frac{\rho_0 r^2 \sin\varphi \, d\varphi \, d\phi \, dr}{\left\{1 - \frac{(\omega' r \sin\varphi \cos\phi)^2}{c^2} - \frac{(v + \omega' r \sin\varphi \sin\phi)^2}{c^2}\right\}^{1/2}} \tag{2.48}$$

Now take a second relativistic approximation again retaining only second order terms. Thus

$$m_{ele} = \rho_0 r^2 \sin\varphi \left\{1 + \frac{1}{2c^2}\left(v^2 + 2v\omega' r \sin\varphi \sin\phi + \omega'^2 r^2 \sin^2\varphi\right)\right\} d\phi \, d\varphi \, dr \tag{2.49}$$

Integrating (2.49) with respect to ϕ gives the energy mass of the toroid

$$m_{tor} = \left\{\int_0^{2\pi}\left(\rho_0 r^2 \sin\varphi + \frac{\rho_0 v^2 r^2 \sin\varphi}{2c^2} + \frac{\rho_0 \omega'^2 r^4 \sin^3\varphi}{2c^2} + \frac{\rho_0 v \omega' r^3 \sin^2\varphi \sin\phi}{c^2}\right) d\phi\right\} d\varphi \, dr \tag{2.50}$$

Which integrates to

$$m_{tor} = \left(2\pi\rho_0 r^2 \sin\varphi + \frac{\pi\rho_0 v^2 r^2 \sin\varphi}{c^2} + \frac{\pi\rho_0 \omega' r^4 \sin^3\varphi}{c^2}\right) d\varphi \, dr \tag{2.51}$$

Integrating (2.51) with respect to φ gives the energy mass of the spherical shell, thus

$$m_{sh} = \left\{\int_0^{\pi/2}\left(4\pi\rho_0 r^2 \sin\varphi + \frac{2\pi\rho_0 v^2 r^2}{c^2}\sin\varphi + \frac{2\pi\rho_0 \omega'^2 r^4}{c^2}\sin^3\varphi\right) d\varphi\right\} dr \tag{2.52}$$

which becomes

$$m_{sh} = \left(4\pi\rho_0 r^2 + \frac{2\pi\rho_0 v^2 r^2}{c^2} + \frac{4}{3}\frac{\pi\rho_0 \omega'^2 r^4}{c^2}\right) dr \tag{2.53}$$

Finally integrating (2.53) with respect to r gives the energy mass of the sphere thus

$$m_{sph} = 4\pi\rho_0 \int_0^{\Gamma}\left(r^2 + \frac{2\pi\rho_0 v^2}{c^2} r^2 + \frac{4}{3}\frac{\pi\rho_0 \omega'^2}{c^2} r^4\right) dr \tag{2.54}$$

which yields

$$m_{sph} = \frac{4}{3}\pi\rho_0 \Gamma^3 \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{1}{5}\frac{\omega'^2 \Gamma^2}{c^2}\right) \tag{2.55}$$

Converting this to a standard relativistic mass equation via an approximate binomial contraction gives

$$m_{sph} \cong \frac{m_0}{\left(1 - \frac{v^2}{c^2} - \frac{2}{5}\frac{\omega'^2 \Gamma^2}{c^2}\right)^{1/2}} \cong \frac{m_0}{\left(1 - \frac{v^2}{c^2} - \frac{\omega'^2 \Gamma_{gyr}^2}{c^2}\right)^{1/2}} \tag{2.56}$$

For the purpose of determining the energy mass of a spinning spherical body also possessing a linear motion, it therefore appears that, in a second order relativistic approximation, the two separate velocities, each at their own individual mass effective dimension, are simply added together as orthogonal vectors.

Note that because of this combination of the two velocities, the limits to which each can be theoretically increased is reduced. In particular, if v is large enough, it would not be possible to increase ω such that $\omega\Gamma \to c$.

3 Concluding Remarks.

By far the most significant result to emerge from this investigation, is the manner in which the mass is relativistically increased as the spin rate increases. It allows the spin circumference to achieve terminal velocity while the mass is still finite. A further increase of spin rate would result in the spin circumference exceeding the terminal velocity in D_0, which would contravene the primary criterion of existence in the Domain. To avoid this it is proposed that the surface of the spinning body that has acquired terminal velocity emits a spectrum of radiation the energy of which equates to that attempting the further increase in spin rate. This proposed effect is based upon the results of a study into the nature of de Broglie matter waves in D_0, to be published in the future.

There are two possible implications of this result in Cosmology, and one in the theory of atomic structure. First the cosmological. Of the two possible implications, the most plausible is as the cause of Pulsar radiation. When a large star reaches the end of its life in the main sequence, it starts to gravitationally collapse in on itself. As it does so, due to the law of conservation of angular momentum, its spin rate will continually increase. If the initial spin rate is high enough, it may increase during the collapse such that the spin velocity at the circumference approaches the limiting value in D_0. It is unlikely that the collapse would produce a perfectly spherical body, and there would most likely be a number of high spots some of which would be on the spin circumference. It would be the most prominent of these that would attain terminal velocity first, and thereby start the radiative loss of energy. Any other astronomical body some distance away but lying on the spin plane of rotation would then experience a series of closely spaced radiation pulses from the Pulsar, as the radiating prominence swept round.

To give an idea of scale, if the Pulsar pulse rate was 4 pulses/sec. it would be of the order of 150,000 km in diameter, or about the size of Jupiter, and is therefore getting close to the estimated size of Neutron stars.

This phenomenon will be tempered by the degree with which any likely star candidate possesses a translational velocity.

The second possible cosmological implication concerns Quasars. However, by comparison with Pulsars, there is the considerable question of scale. The radiant output of Quasars is so vast, that even with the manner of generation proposed here, effectively the direct conversion of matter to radiant energy, it is most unlikely that it is a collapsed star that is acting as the generator. The most likely candidate would therefore have to be a gravitationally collapsed galactic core. Even so, to generate the power that these objects exhibit, a considerable amount of the surface area would have to be the radiative medium. All this probably means that the mechanism proposed here is not the one responsible for Quasar emissions. Nevertheless, until the correct mechanism is identified, the one discussed here should not be entirely discounted.

The final possible manner in which this mechanism may be manifested, is in the emission of spectra by an electron when making an orbital transition in the atomic structure of matter. This scenario is, however contrary to the modern Quantum Mechanical Theory of atomic structure, in which the electron is represented by a probabilistic wavefunction, and is not purported to actually "spin". However, in a series of papers to be published in the future, in which the original Bohr/Sommerfeld "Old Quantum Theory" of atomic structure is to be resurrected, this matter will investigated in depth.

All other results in this paper are largely of mathematical interest only.

REFERENCES.

[1] P.G.Bass, *The Special Theory of Relativity - A Classical Approach.* Apeiron (4) Vol.10 October 2003.

[2] P.G.Bass, *Gravitation - A New Theory.* Apeiron (4) Vol.10. October 2003.

[3] Albert Einstein, *The Special and general Theories.* University Paperbacks, 1960.

Note: The first two of the above references also appear on the authors website :-

www.relativitydomains.com