THE APPLICABILITY OF THE LAWS OF MOTION WITHIN A RELATIVISTIC DOMAIN.

Peter G.Bass

ABSTRACT.

This paper reviews the relativistic applicability of both Sir Isaac Newton's three general laws of motion, and Johannes Kepler's three laws of planetary motion, within their applicable relativistic domains, D_0, (Pseudo-Euclidean Space-Time), and D_1, (Gravitational Space-Time).

1 Introduction.

The work of Tycho Brahe, the Danish astronomer, in making the most precise positioning of the planets of the day, the late sixteenth century, provided the foundation upon which the German astronomer Johannes Kepler developed his three laws of planetary motion.
Subsequently, the English mathematician, Sir Isaac Newton, showed that these laws were perfectly compatible with each planet possessing an acceleration towards the sun which was inversely proportional to the square of the distance from it. He was able to generalise this as being of the same nature as that experienced by objects in free fall close to the Earth's surface. As a result of this and other related observations, Newton deduced his three general laws of motion which were published in 1687 in his great work Philosophiae Naturalis Principia Mathematica.

The laws of motion of both Newton and Kepler are perfectly applicable today for whatever mechanical purpose may be pursued, including the space flight projects that have so far been undertaken. It is only in astronomical and cosmological aspects, in which motion at velocities approaching that of light, and/or over very long periods of time, that relativistic effects become significant and which may therefore affect their veracity.

This paper examines the applicability of both Newton's and Kepler's laws under these latter conditions. Because Kepler's laws are to do solely with motion due to gravity, the applicability of them will only be considered within the Gravitational Domain D_1. Newton's laws however, are to do with artificially induced motion as well as that due to gravitation, (free fall within the gravitational field), and therefore, his laws will be examined in both D_0, Pseudo-Euclidean Space-Time, and D_1, as appropriate. Because the three laws of Newton are the more general, they are considered first.

For a full understanding of the relativistic theory as presented in this paper, it is important to read the applicable sections of [2], [3], [6] and [7] first. Finally, to avoid multiple cross referencing, all equations referred to in the references will be repeated herein and their parameters re-defined.

2 Newton's Laws of Motion - Relativistic Domain Theory Applicability.

2.1 Within the Relativistic Domain D_0, (Pseudo-Euclidean Space-Time).

It is important to note that in the discussion of these laws, all applied forces referred to are defined as purely spatial. The corresponding temporal forces generated as a result of the rotation of the Existence Velocity vector in the X_0 - R plane, cancel each other as is demonstrated in [6]. However, it is still necessary to perform all analytical derivations here within the full spatial-temporal dimensions in order to include the correct mass rate effects.

2.1.1 **Newton's First Law.**

In [1] Newton's first law is stated as follows:-

> "Every body preserves in its state of rest or of uniform motion in a straight line except insofar as it is compelled to change that state by impressed forces."

In [4] and [5] this has been given mathematical form in spatial Cartesian co-ordinates thus,

$$\frac{d^2x}{dt^2}\ ;\ \frac{d^2y}{dt^2}\ ;\ \frac{d^2z}{dt^2} = 0 \tag{2.1}$$

However, by way of introducing the mass, it is considered that the following is a more suitable expression of this law,

$$m_0\frac{d\boldsymbol{v}}{dt} = 0 \tag{2.2}$$

Where,

$\boldsymbol{v}$ = The velocity vector.

m_0 = The rest mass

References [4] and [5] have discussed both four dimensional and relativistic modifications to (2.1), but the important concept of Existence Momentum is absent from these discussions. Examination of Existence Momentum is considered necessary in the relativistic evaluation of these laws because it involves the mass of the body and in relativistic motion the mass is a variable.
The spatial-temporal Existence Momentum of a mass in D_0 is, in [6], Eq.(3.1) given by,

$$\mathrm{M} = m\mathrm{V} = m\left\{\boldsymbol{v} + \boldsymbol{j}c\left(1-\frac{v^2}{c^2}\right)^{1/2}\right\} \tag{2.3}$$

Where

M = The spatial-temporal Existence Momentum vector.

V = The spatial-temporal Existence Velocity vector.

m = The energy mass.

$\boldsymbol{j}$ = The temporal unit vector.

c = The spatial terminal velocity in D_0 , ($\approx$ the velocity of light).

Accordingly, in studying the Relativistic Domain Theory applicability of this law, the relationship to be analysed is,

$$\frac{d\mathrm{M}}{dt} = 0 \tag{2.4}$$

The criteria satisfying (2.4) are determined via the following simple process.
The energy mass m, of a body in motion, is given by [6], Eq.(3.6), viz.

$$m = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \tag{2.5}$$

Where

v = The magnitude of $\boldsymbol{v}$.

Inserting (2.5) into (2.3) gives,

$$\mathrm{M} = m_0 \left\{ \frac{v}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} + jc \right\} \tag{2.6}$$

so that

$$\frac{d\mathrm{M}}{dt} = \frac{m_0 \dot{v}}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} + \frac{m_0 v \dot{v}}{c^2 \left(1 - \frac{v^2}{c^2}\right)^{3/2}} \tag{2.7}$$

and is all spatial.
Eq.(2.7) shows that the only requirement for (2.4) to be met is that the spatial velocity be constant in both magnitude and direction. This is identical to the condition of (2.2) and therefore Newton's first law, as stated above, is without change, fully applicable to relativistic motion within D_0. Consequently, its mathematical expression, (2.2), is also fully applicable. However, it will become apparent from later analysis in relation to the third law, that (2.4) may be a more appropriate expression.
It is also of interest to note that, from (2.6), temporal momentum is always constant irrespective of the state of spatial motion.

2.1.2 <u>**Newton's Second Law.**</u>

In [1] Newton's second law of motion is stated as follows,

> "Change of momentum is proportional to the impressed force and takes place along the line of action of that force".

Mathematically it is stated in [1] to be represented by the equation,

$$\frac{d}{dt}(mv) \propto F \tag{2.8}$$

which assumes that the mass is a variable. The more generally accepted non-relativistic relationship, also quoted in [1], is

$$m\frac{dv}{dt} \propto F \tag{2.9}$$

in which the mass is implicitly defined as a constant, (of course this relationship is well known to be an equality rather than a proportionality).
In [4] and [5] et al the relativistic modification of this law is discussed. However, once again, despite (2.8) above, none of these discussions deal with Existence Momentum or, more importantly, in relativistic curvi-linear motion, that the applied force vector, the spatial acceleration vector and the spatial velocity vector, all lie in different directions, (see [6]).
The relativistic mathematical expression to be analysed here, is given simply by,

$$\frac{d\mathrm{M}}{dt} = F \tag{2.10}$$

Where

$\boldsymbol{F}$ = An applied spatial force vector.

Eq.(2.10) was analysed in some depth in [6] where the statement above concerning the non-alignment of the force, acceleration and velocity vectors was demonstrated as, viz. [6], Eq.(3.24),

$$\tan\psi = \frac{dv_y/dt}{dv_x/dt} = \frac{\sin\xi - {}^{v^2}\!/_{c^2}\sin\eta\cos(\xi-\eta)}{\cos\xi - {}^{v^2}\!/_{c^2}\cos\eta\cos(\xi-\eta)} \tag{2.11}$$

Where

$\psi =$ The angle between the acceleration vector and the X axis.

$\xi =$ The angle between the applied force vector and the X axis.

$\eta =$ The angle between the velocity vector and the X axis.

The geometry is depicted in [6], Fig.4.1 and clearly, from (2.11) these three vectors are only co-incident when,

$$\xi = \eta \tag{2.12}$$

i.e. for pure rectilinear motion.
As a result of the above discussion it is clear that the definition of Newton's second law is not adequate to rigorously cover full relativistic curvi-linear motion. To do so requires a re-statement thus,

> "Change of momentum is proportional to the impressed force and takes place such that the direction of the acceleration vector is proportional to the directional vectors of the impressed force and the velocity".

which clearly covers both rectilinear and curvi-linear motion. The mathematical relationship applicable to this revised statement is that of (2.10) as augmented by (2.11).

2.1.3 <u>**Newton's Third Law.**</u>

In [1] Newton's third law is stated as follows,

> "Action and reaction are always equal and opposite; that is to say, the actions of two bodies upon one another, are equal and directly opposite"

Mathematically, this law is also represented by (2.9) because the term on the LHS of this relationship represents the reaction to the applied force.
In relativistic motion it can be stated without further review that this law, as defined above, does not apply. This is because it takes no account of the spatial mass rate reaction terms. Spatial mass rate reactions, for artificially accelerated motion is discussed in [6], Section 4. It is here that the non-applicability of this law is most apparent. As an example, [6], Eq.(4.31) and [6], Eq.(4.32) show that a small reaction force is generated at right angles to the applied force due to the combination of the mass rate effect and the velocity. This effect is demonstrated in [6] by the relationships,

$$\begin{aligned} \frac{dv_x}{dt} &= -\frac{F_y v^2}{m_0 c^2}\left(1-\frac{v^2}{c^2}\right)^{1/2}\sin\eta\cos\eta \\ m_{ax} &= 0 \end{aligned} \tag{2.13}$$

Where

dv_x/dt = Acceleration in the X direction.
F_y = Applied force in the Y direction.
v = Magnitude of the total velocity vector.
c = Terminal velocity in D_0, ($\approx$ the velocity of light).
η = The angle between v and the X axis.
m_{ax} = Inertial mass in the X direction.

It is precisely this effect that is the cause of the non-alignment of the force, acceleration and velocity vectors discussed in the relation to the second law above.
For applicability to motion in a relativistic domain, this law has to be completely restated as follows,

> "To every action causing acceleration of a mass there are always two reactions. The mass reaction will always act in opposition along the acceleration vector, while the mass rate reaction will always act in opposition along the velocity vector.

Note that this automatically covers rectilinear motion where all force and motion vectors are co-incident. A formula specifically describing this law in relativistic terms is then,

$$m\frac{dv}{dt} + v\frac{dm}{dt} = F \tag{2.14}$$

This is evident from [6], Eqs.(3.10), (3.11), (3.12) and (3.13) and the associated discussion.

2.2 Consolidation.

The original three laws as expressed by (2.2) and (2.9), can be consolidated into one, that of (2.9) and the first law obtained from (2.9) by putting $\boldsymbol{F}$ to zero. In the relativistic case, this consolidation can only strictly be maintained for the first and third laws, i.e. (2.10), wherein the first relativistic law is obtained by putting $\boldsymbol{F}$ to zero, {note that for the third law, in (2.14) the LHS is simply the time differential of (2.10)}. In addition to (2.10), the second relativistic law also requires the more detailed but explicit relationship of (2.11).

2.3 The Incorporation of Gravitation - (The Gravitational Space-Time D_1).

All of the subject analysis in the above Section has been carried out in the relativistic domain D_0, (Pseudo-Euclidean Space-Time). The three laws of Newton in their original non-relativistic form, and in their modified relativistic form, have all been expressed within the characteristics of that domain. However, all motion of whatever kind takes place within a gravitational field, the domain D_1, and the final and proper form of the relativistic laws should be expressed within such a domain.
Gravitational motion, both free and as augmented by artificially applied forces, were analysed in detail in [2] and [7] respectively. Accordingly, from these it is a simple matter to combine [2], Eq.(3.17) and (2.10) to give,

$$\frac{d\mathrm{M}}{d\tau} = F - mc^2 u\frac{du}{d\sigma} \tag{2.15}$$

where co-ordinates of space and time, σ and τ, are those of D_1, Gravitational Space-Time, and u is the temporal rate of D_1 with respect to D_0. The relationship of (2.15) is in fact quoted in [7], {[7]. Eq.(3.16)} and analysed in depth there.

Eq.(2.15) is the most general relationship which, when appropriately expanded, expresses Newton's third law of relativistic motion within either the Gravitational Space-Time D_1, or the Pseudo-Euclidean Space-Time D_0, in the latter by putting $u = 1$, so that $du/d\sigma = 0$ and $\tau = t$.
In the former domain however, the equivalent first law has to be restated because in putting $\boldsymbol{F}$ to zero in (2.15) leaves,

$$\frac{d\mathrm{M}}{d\tau} = -mc^2 u \frac{du}{d\sigma} \tag{2.16}$$

i.e. free gravitational motion as derived in [2], Eq.(3.17). The difference between (2.16) and (2.4) is very clear and expresses the fact that the natural state of existence in D_0 is different from that in D_1. In D_0 the natural state of existence is to be spatially at rest, while in D_1 it is to spatially accelerate towards the gravitational source. This was fully demonstrated and explained in [2], [6], and [7].
Within D_1 therefore, an appropriate first law of (gravitational) motion would be stated as follows,

> "Within a gravitational field, every body preserves in its state of acceleration towards the gravitational source, except insofar as it is compelled to change that state by artificially impressed forces."

Newton's second law of motion within D_1 remains in its relativistic form as stated above, because the addition of the gravitational effect merely augments the artificial force term already present. Mathematical representation is by (2.15) and (2.11), the latter as modified with the change to gravitational spatial and temporal parameters.

3 Kepler's Laws of Planetary Motion - Relativistic Domain Theory Applicability.

3.1 Kepler's First Law.

In [1], Kepler's first law is stated as follows,

> "The planets describe ellipses with the Sun situated at a focus"

In [1], after analysis, this is slightly extended, to show that the planets describe conic sections about the Sun at one focus. It is well known that all gravitationally generated central orbits, in non-relativistic form, open or closed, describe conic sections given by,

$$\frac{L}{r} = 1 + \varepsilon \cos(\phi - \phi_0) \tag{3.1}$$

Where

L = The semi-latus rectum.

ε = Eccentricity.

r = The radius vector to the focal point.

ϕ = The radial angle to the major axis.

ϕ_0 = A constant, (initial condition).

In the Relativistic Domain D_1, it is shown in [2] Section 5, that a central orbit is also a conic section described thus, [2], Eq.(5.11),

$$\mu = \frac{1}{L}\left\{ 1 + \varepsilon \cos(\phi - \Omega)\right\} \tag{3.2}$$

Where in [2], Eq.(5.6)

$$\mu = \frac{1}{\sigma} \tag{3.3}$$

and in [2], Eq.(4.18)

$$\sigma = r + \alpha \tag{3.4}$$

and where in [2], Eq.(5.34),

$$\Omega \cong \frac{3\alpha\mu_0}{1+\varepsilon}\phi + \frac{\alpha\mu_0 \sin\phi}{1+\varepsilon} \tag{3.5}$$

In (3.2), (3.3), (3.4) and (3.5) the terms are defined thus,

Ω = Orbit precession angle.

α = Gravitational radius of the source.

μ_0 = Initial condition of μ, (Inverse distance to orbit focus at perihelion).

The derivation of (3.2), (3.3), (3.4) and (3.5) is shown in [2] and in [3] it is shown that (3.4) is a relativistic effect within the gravitational source.
Because Ω is a variable, a precise re-statement of Kepler's first law, specifically for relativistic effects would therefore need to be as follows,

> "All gravitationally generated central orbits describe conic sections with the gravitational source situated at one focus, and in which precession of the orbit is a function of both the radial angle to the major axis and the eccentricity of the basic curve."

3.2 Kepler's Second Law.

In [1], Kepler's second law is stated as follows,

> "The radius vector joining the Sun with a planet describes equal areas in equal times, i.e. the rate of description of sectorial area is constant".

Mathematically this is stated as,

$$\frac{dz}{dt} = \frac{h}{2} = \frac{\omega r^2}{2} = \frac{\omega_0 r_0^2}{2} \tag{3.6}$$

Where,

z = Swept area.

h = Swept area constant.

ω = Angular rate

r = Radial distance.

ω_0 and r_0 are initial conditions.

In [2] it is shown, [2], Eq.(5.4), that the relativistic version of the parameter h is given by,

$$h = \frac{\omega\sigma^2}{\left(1 - \frac{\dot{\sigma}^2}{c^2u^2} - \frac{\omega^2\sigma^2}{c^2}\right)^{1/2}} = \frac{\omega_0^2\sigma_0^2}{\left(1 - \frac{\omega_0^2\sigma_0^2}{c^2}\right)^{1/2}} \tag{3.7}$$

and thus the relativistic version of (3.6) becomes,

$$\frac{dz}{d\tau} = \frac{h}{2} = \frac{\omega_0\sigma_0^2}{2\left(1 - \frac{\omega_0^2\sigma_0^2}{c^2}\right)^{1/2}} \tag{3.8}$$

Where,

τ = Time in D_1.

and other terms are as defined above.
Kepler's second law therefore holds in Relativistic Domain Theory with the relationship modification as shown in (3.8).

3.3 Kepler's Third Law.

In [1], Kepler's third law is stated as follows,

> "The cubes of the mean distances of the planets from the Sun are proportional to the squares of their times of revolution, i.e. if $2a$ is the major axis of the elliptic orbit and t is the periodic time, then $t^2 \propto a^3$".

In [1] the mathematical relationship for this law is derived and quoted as,

$$\tau = \frac{2\pi a^{3/2}}{\sqrt{\mu}} \tag{3.9}$$

Where,

$2a$ = The major axis.

τ = The periodic time, ($\equiv$ t in the definition).

μ = A constant related to the gravitational force exerted by the source at the focus, (it is not the same as the parameter μ used in this series of papers).

In the Relativistic Domain D_1, to determine whether this law holds, the equation of motion applicable to a central orbit is used as the starting point. This is given by [2]. Eq. (3.17), viz.

$$\ddot{\sigma} = -c^2 u \frac{du}{d\sigma} + 2\frac{\dot{\sigma}}{u}\frac{du}{d\sigma} + u^2\omega^2\sigma \tag{3.10}$$

Where

u = The temporal rate at σ.

ω = The angular rate.

From [2], Eqs. (4.6) and (4.7)

$$u = \left(1 - \frac{2\gamma m_G}{\sigma c^2}\right)^{1/2} \tag{3.11}$$

So that in (3.10)

$$-c^2 u \frac{du}{d\sigma} = -\frac{\gamma m_G}{\sigma^2} \tag{3.12}$$

as derived in [2], Eq. (4.4) and where

γ = Newton's constant of proportionality.

m_G = The mass of the gravitational source.

Inserting (3.12) into (3.10) gives,

$$\ddot{\sigma} = -\frac{\gamma m_G}{\sigma} + 2\frac{\dot{\sigma}}{u}\frac{du}{d\sigma} + u^2\omega^2\sigma \tag{3.13}$$

In (3.13) over one complete orbit, the mean values of $\dot{\sigma}$ *and* $\ddot{\sigma}$ will be zero, σ will become σ_{OR}, ω will become ω_{OR}, and u becomesu_{OR} where,

σ_{OR} = The mean value of σ over one complete orbit.

ω_{OR} = The mean value of ω over one complete orbit.

u_{OR} = The mean value of u over one complete orbit.

so that (3.13) then becomes,

$$u_{OR}^2 \omega_{OR}^2 \sigma_{OR} = \frac{\gamma m_G}{\sigma_{OR}^2} \tag{3.14}$$

u_{OR} is determined simply thus. The temporal rate at σ is given by [2], Eq. (4.6), viz,

$$u = \left(1 - \frac{2\alpha}{\sigma}\right)^{1/2} \tag{3.15}$$

so that

$$u_{OR} = \left(1 - \frac{2\alpha}{\sigma_{OR}}\right)^{1/2} \tag{3.16}$$

ω_{OR} is determined as follows. In one complete orbit the angle rotated through is

$$\phi_{OR} = 2\pi - \Omega_{OR} \tag{3.17}$$

Where

Ω_{OR} = The single orbit perihelion precession angle.

From (3.5) this is given by

$$\Omega_{OR} = \frac{6\pi\alpha\mu_0}{1+\varepsilon} \tag{3.18}$$

and therefore the mean value of the angular rate over one orbit is,

$$\omega_{OR} = \frac{\phi_{OR}}{\tau_{OR}} = \frac{2\pi}{\tau_{OR}}\left(1 - \frac{3\alpha\mu_0}{1+\varepsilon}\right) \tag{3.19}$$

Where

τ_{OR} = The periodic time of the orbit.

Here μ_0 is as defined in this series of papers.
Substitution of (3.16) and (3.19) into (3.14) then gives

$$\frac{\gamma m_G}{\sigma_{OR}^2} = \frac{4\pi^2}{\tau_{OR}^2}\left(1 - \frac{2\alpha}{\sigma_{OR}}\right)\sigma_{OR}\left(1 - \frac{3\alpha u_0}{1+\varepsilon}\right)^2 \tag{3.20}$$

Re-arrangement into Kepler's format then yields,

$$\frac{\sigma_{OR}^2(\sigma_{OR} - 2\alpha)}{\tau_{OR}^2} = \frac{\gamma m_G (1+\varepsilon)^2}{4\pi^2(1+\varepsilon-3\alpha u_0)^2} \tag{3.21}$$

Eq.(3.21) shows that, irrespective of the actual value of σ_{OR}, Kepler's third law does not quite hold under relativistic conditions. The reason being the presence of the -2α term on the LHS of (3.21) and the -3$\alpha\mu_0$ term on the RHS.
Compounding this, there is a difference in the mean value of the radius vector, σ_{OR}. This is shown as follows.
Inserting the classical value for L into (3.2) and re-arranging gives,

$$\sigma = \frac{p\left(1-\varepsilon^2\right)}{1+\varepsilon\cos(\phi-\Omega)} \tag{3.22}$$

Where,

$2p$ = The major axis length of the orbit.

Now, when $\phi = 0$, $\Omega = 0$ and (3.22) reduces to,

$$\sigma|_{\phi=0} = p(1-\varepsilon) \tag{3.23}$$

and when $\phi = \pi$, from (3.5), $\Omega = \dfrac{3\pi\alpha\mu_0}{1+\varepsilon} = \dfrac{\Omega_{OR}}{2}$ and so (3.22) becomes

$$\sigma|_{\phi=\pi} = \frac{p\left(1-\varepsilon^2\right)}{1-\varepsilon\cos\left(\Omega_{OR}/2\right)} \tag{3.24}$$

Therefore, from (3.23) and (3.24)

$$\sigma_{OR}\Big|_{\phi=0}^{\pi} = \frac{p(1-\varepsilon)\left[1+\frac{\varepsilon}{2}\left\{1-\cos\left(\Omega_{OR}/2\right)\right\}\right]}{\left\{1-\varepsilon\cos\left(\Omega_{OR}/2\right)\right\}} \tag{3.25}$$

As both halves of the orbit are identical, (3.25) also applies over the complete orbit.
Eq.(3.25) shows that the difference between the mean values of the radius vectors in Kepler's original third law and the relativistic version is solely due to the orbit precession angle.
Approximation to the non-relativistic version, Kepler's original, is obtained by allowing $c \to \infty$ so that both α and Ω_{OR} become zero, which results in (3.25) becoming

$$\sigma_{OR} = p \tag{3.26}$$

so that (3.21) then becomes

$$\frac{p^3}{\tau_{OR}^2} = \frac{\gamma m_G}{4\pi^2} \tag{3.27}$$

as in (3.9).

The combination of Eqs.(3.21) and (3.25), although not equivalent to Kepler's third law is very close to it and represents the relationship for an elliptic orbit. Putting $\varepsilon = 0$ produces the relationship for a circular orbit.
In view of the smallness of the discrepancy of (3.21) and (3.25) from (3.9), a re-statement of this law for a relativistic planetary orbit is unnecessary provided the modified relationships are used as appropriate

3.4 Summary.

A summary of the mathematical representations of all the laws as presented here is shown below in tabular form.

LAW		NON - RELATIVISTIC	RELATIVISTIC $\mathbf{D}_0$	RELATIVISTIC $\mathbf{D}_1$
Newton	1st	$m_0\dfrac{dv}{dt}=0$	$\dfrac{d\mathrm{M}}{dt}=0$	$\dfrac{d\mathrm{M}}{d\tau}=-mc^2u\dfrac{du}{d\sigma}$
	2nd	$m_0\dfrac{dv}{dt}=F$	$\dfrac{d\mathrm{M}}{dt}=F$ *and* $\tan\psi=\dfrac{\sin\xi-\frac{v^2}{c^2}\sin\eta\cos(\xi-\eta)}{\cos\xi-\frac{v^2}{c^2}\cos\eta\cos(\xi-\eta)}$	$\dfrac{d\mathrm{M}}{d\tau}=F-mc^2u\dfrac{du}{d\sigma}$ *and* $\tan\psi=\dfrac{\sin\xi-\frac{\nu^2}{c^2}\sin\eta\cos(\xi-\eta)}{\cos\xi-\frac{\nu^2}{c^2}\cos\eta\cos(\xi-\eta)}$
	3rd	$m_0\dfrac{dv}{dt}=F$	$\dfrac{d\mathrm{M}}{dt}=F$	$\dfrac{d\mathrm{M}}{d\tau}=F-mc^2u\dfrac{du}{d\sigma}$
Kepler	1st	$\dfrac{L}{r}=1+\varepsilon\cos(\phi-\phi_0)$	N/A	$\mu=\dfrac{1}{L}\{1+\varepsilon\cos(\phi-\Omega)\}$
	2nd	$\dfrac{dz}{dt}=\dfrac{\omega_0 r_0^2}{2}$	N/A	$\dfrac{dz}{d\tau}=\dfrac{\omega_0\sigma_0^2}{2\left(1-\frac{\omega_0^2\sigma_0^2}{c^2}\right)^{1/2}}$
	3rd	$\dfrac{p^3}{\tau_{OR}^2}=\dfrac{\gamma m_G}{4\pi^2}$	N/A	$\dfrac{\sigma_{OR}^2(\sigma_{OR}-2\alpha)}{\tau_{OR}^2}=\dfrac{\gamma m_G(1+\varepsilon)^2}{4\pi^2(1+\varepsilon-3\alpha u_0)^2}$

Note that the parameters in the original laws as quoted in [1], have, in the above table, been co-ordinated with those as used in this series of papers. Also in the second expression for the gravitational version of Newton's second law, $\nu=d\sigma/d\tau$.

4 Concluding Remarks.

The applicability of Newton's three general laws of motion within a relativistic Domain are affected primarily by the influence of the mass rate. This parameter modifies both the second and third laws. These modifications are such that they disturb the degree of consolidation of the original three laws, to just the relativistic first and the third. It could be said that (2.10) also incorporates the second relativistic law, thus maintaining the consolidation of all three, but it is considered that (2.10) must be augmented by (2.11) to provide a more explicit mathematical description of the second law in D_0.
When the effects of gravity are introduced, the first law is no longer applicable. This is because of the different natural states of existence within the two Domains, Pseudo-Euclidean Space-Time and the Gravitational Space-Time. The first law has thereby to be re-stated and effectively becomes a first law of gravitational motion. The original first law, either relativistic or otherwise, is then only applicable in the (assumed) complete absence of a gravitational field.

The fact that Kepler's three laws of planetary motion have been found to be valid under relativistic conditions with only very minor modification is not surprising. The reason is that the nature of planetary central orbits, i.e. their orbital velocities, are such as to be only minimally subject to relativistic effects.

REFERENCES

[1] R.H.Aitkin, *Classical Dynamics,* Heinemann, (1959).

[2]* P.G.Bass, *Gravitation - A New Theory,* Apeiron, (4), Vol. 10, (2003).

[3]* P.G.Bass, *Generation of the Acceleration Potential and the Time Dilatation Effect,* Apeiron, (1), Vol. 11, (2004).

[4] E.E.Condon/Hugh Odishaw, *Handbook of Physics,* McGraw Hill, (1967).

[5] Richard Feynman, *Six Not So Easy Pieces,* Perseus Books, (1997).

[6]* P.G.Bass, *The Special Theory of Relativity - A Classical Approach,* Apeiron (4), Vol. 10, (2003).

[7]* P.G.Bass, *Further Kinetics of Gravitational Motion,* Apeiron (1), Vol. 11, (2004).

* - Also on www.relativitydomains.com.