THE SPATIAL - TEMPORAL DISTRIBUTION OF ENERGY IN THE RELATIVISTIC MOTION OF PONDERABLE MATTER.

P.G.Bass.

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ABSTRACT.

This short paper presents an alternative derivation of the spatial - temporal distribution of both the kinetic, and matter energies of a mass of ponderable matter subject to spatial acceleration in the Relativistic Domain $\mathbf{D_0}$. The results are shown in Relativistic Domain nomenclature, and also as functions of Euclidean time to demonstrate limiting values in that parameter.

CONTENTS.

1.0 Introduction.

In [2], via a derivation of Einstein's energy - momentum equation from a consideration of Max Planck's quantum energy, and Louis de Broglie's quantum momentum hypotheses, it was shown that the energy of particle matter has a spatial - temporal existence. A short ensuing analysis derived the two components of that existence.

In this paper, from a different approach identical to that in [1], the same effects are investigated for the accelerated motion of ponderable matter by an external artificial force. This investigation is effected via a consideration of the manner in which kinetic energy is added to such a mass as its spatial velocity increases.
This is followed by a re-arrangement of some of the pertinent relationships into a form explicitly involving Pseudo-Euclidean time. These then enable an exact determination of limiting values in such terms.

In the interests of brevity, unless necessary for complete clarity, a parameter will only be defined in this paper if it has not already been so in [1] or [2] with which familiarity is assumed.

2.0 The Spatial - Temporal Distribution of Energy of Ponderable Matter Under Spatial Acceleration.

Considering the relativistic nature of the accelerated motion of ponderable matter, as analysed in [1], the manner in which kinetic energy is added to the matter energy vector, is shown in Fig. 2.1 below, and described subsequently.

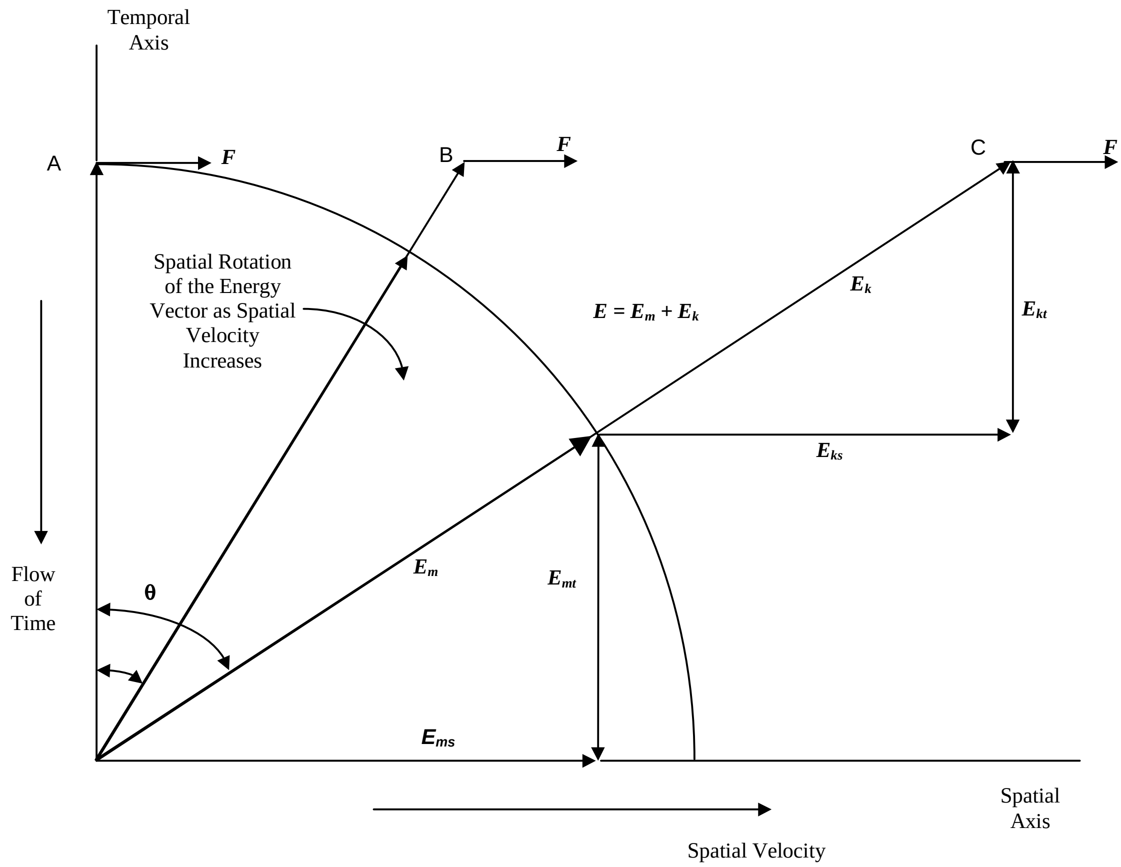

Fig. 2.1 - Spatial - Temporal Energy Vectors.

In Fig. 2.1 the terms are defined as follows:-

θ		is the Spatial - Temporal Angle of $\boldsymbol{E}$.
$\boldsymbol{E} =$	$\boldsymbol{mc^2}$	is Einstein's relationship for the total energy of the mass.
$\boldsymbol{E_m} =$	$\boldsymbol{m_0c^2}$	is the rest mass matter energy of the mass.
$\boldsymbol{E_{ms}} =$	$\boldsymbol{E_m \sin\theta}$	is the spatial projection of $\boldsymbol{E_m}$.
$\boldsymbol{E_{mt}} =$	$\boldsymbol{E_m \cos\theta}$	is the temporal projection of $\boldsymbol{E_m}$.
$\boldsymbol{E_k} =$	$\boldsymbol{mc^2 - m_0c^2}$	is the spatial - temporal kinetic energy of the mass.
$\boldsymbol{E_{ks}} =$	$\boldsymbol{E_k \sin\theta}$	is the spatial projection of $\boldsymbol{E_k}$.
$\boldsymbol{E_{kt}} =$	$\boldsymbol{E_k \cos\theta}$	is the temporal projection of $\boldsymbol{E_k}$.

Referring to Fig. 2.1, as the mass is spatially accelerated by the force $\boldsymbol{F}$ from A, (at rest), to B and then C, kinetic energy is continually added to the total energy vector as it rotates into the spatial dimension through the spatial - temporal angle θ. The original matter energy, $\boldsymbol{E_m}$, remains constant while the kinetic term, $\boldsymbol{E_k}$, continues to grow. As the spatial velocity increases and the energy vector rotation continues, as a consequence both the matter and kinetic energy terms clearly exhibit spatial - temporal projections as shown and defined above. Determination of these projected components is then straightforward as follows.
Firstly the matter energy components.

$$E_{ms} = E_m \,\mathbf{sin}\,\theta = m_0c^2\frac{v}{c} = m_0vc$$

$$E_{mt} = E_m \,\mathbf{cos}\,\theta = m_0c^2\left(1-\frac{v^2}{c^2}\right)^{1/2} \tag{2.1}$$

so that in Relativistic Domain nomenclature, (as in [1]),

$$E_m = E_{ms} + jE_{mt} = m_0vc + jm_0c^2\left(1-\frac{v^2}{c^2}\right)^{1/2} \tag{2.2}$$

Secondly the kinetic energy components

$$E_{ks} = E_k \,\mathbf{sin}\,\theta = \left(mc^2 - m_0c^2\right)\frac{v}{c} = \left(m - m_0\right)vc$$

$$E_{kt} = E_k \,\mathbf{cos}\,\theta = \left(mc^2 - m_0c^2\right)\left(1-\frac{v^2}{c^2}\right)^{1/2} = m_0c^2\left\{1-\left(1-\frac{v^2}{c^2}\right)^{1/2}\right\} \tag{2.3}$$

and in Relativistic Domain nomenclature

$$E_k = E_{ks} + jE_{kt} = (m - m_0)vc + jm_0c^2\left\{1 - \left(1 - \frac{v^2}{c^2}\right)^{1/2}\right\} \tag{2.4}$$

When the spatial and temporal projected components of (2,2) and (2.4) are summed, the results are

(i) for the total spatial component

$$E_s = E_{ms} + E_{ks} = m_0vc + (m - m_0)vc = mvc \tag{2.5}$$

which is believed to exist in the form of a de Broglie matter wave.

and

(ii) for the total temporal component

$$E_t = E_{mt} + E_{kt} = m_0c^2\left(1 - \frac{v^2}{c^2}\right)^{1/2} + m_0c^2\left\{1 - \left(1 - \frac{v^2}{c^2}\right)^{1/2}\right\} = m_0c^2 \tag{2.6}$$

and consequently the total energy is

$$E = E_s + jE_t = mvc + jm_0c^2 = pc + jm_0c^2 \tag{2.7}$$

The magnitude of which is

$$E = \left(p^2c^2 + m_0^2c^4\right)^{1/2} \tag{2.8}$$

i.e. Einstein's energy - momentum relationship and, here it is clear that the term $\boldsymbol{p}$ in (2.7) and (2.8) is the relativistic spatial momentum of the mass.

The analysis above shows that the same overall results for the spatial-temporal distribution of energy has been obtained from two different approaches:-

(i) In [2], from the quantum energy and momentum hypotheses of Max Planck and Louis de Broglie, and

(ii) In this paper via the manner in which kinetic energy is added to the accelerated mass of ponderable matter in the Relativistic Domain $\mathbf{D_0}$ as defined in [1].

To obtain interpretation of limiting values it will be useful to derive the time dependency of some of the above terms.

3.0 Solutions in Terms of Pseudo-Euclidean Time.

3.1 Spatial Velocity and Spatial Acceleration.

All of the pertinent terms derived in [1] and above include the spatial velocity *v*. It is most useful therefore to determine the time dependency of this parameter first.
Starting from [1] Eq.(3.8), repeated here for convenience

$$F = m_a \frac{dv}{dt} = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} \frac{dv}{dt} \tag{3.1}$$

where *F* is constant. This can be expressed in the form of a simple differential equation, thus

$$\frac{F}{m_0} dt = \frac{dv}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} \tag{3.2}$$

the solution of which is

$$\frac{Ft}{m_0} = \frac{v}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} + k \tag{3.3}$$

The constant of integration, *k*, is zero so that re-arrangement of (3.3) then gives the desired result.

$$v = \frac{c}{\left(1+\frac{m_0^2 c^2}{F^2 t^2}\right)^{1/2}} \tag{3.4}$$

From which it is easily seen that when $t = 0$, $v = 0$, and that *v* can only hypothetically acquire the value of *c* when $t \to \infty$.

The time derivative of (3.4) is

$$\frac{dv}{dt} = \frac{F}{m_0\left(1+\frac{F^2 t^2}{m_0^2 c^2}\right)^{3/2}} \tag{3.5}$$

and clearly this specifically shows that

$$\left.\frac{dv}{dt}\right|_{t=0} = \frac{F}{m_0}$$

$$\left.\frac{dv}{dt}\right|_{t=\infty} = 0 \tag{3.6}$$

the former limit being identical to the classical non-relativistic case.

3.2 Energy.

3.2.1. Total Energy.

Expanding the first relationship in (2.7) gives

$$E = \frac{m_0 vc}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} + jm_0c^2 \tag{3.7}$$

and inserting (3.4) for v then gives simply

$$E = Ftc + jm_0c^2 \tag{3.8}$$

and the limits here regarding t are obvious.

Eq.(3.8) shows that the total energy contained within the accelerated mass is spatially a linear function of time, whilst the temporal component is constant. Clearly this is so because the accelerating force F is all spatial. This result is considered important because in comparison with (2.8)

$$|E| = \left(m_0^2c^4 + p^2c^2\right)^{1/2} = \left(F^2t^2c^2 + m_0^2c^4\right)^{1/2} \tag{3.9}$$

so that clearly $p = Ft$. Also because all terms in (3.8) are positive definite, then $|E|$ can only be a positive quantity and therefore the apparent mathematically permissible negative root of E in (2.8) is excluded as non-existent. This has extreme significance when considered in conjunction with the Klein-Gordon and Dirac equations of quantum electrodynamics.

3.2.2. Matter Energy.

From (2.2) matter energy in spatial - temporal form is

$$E_m = m_0vc + jm_0c^2\left(1 - \frac{v^2}{c^2}\right)^{1/2} \tag{3.10}$$

and substituting (3.4) into this yields

$$E_m = \frac{m_0c^2}{\left(1 + \frac{F^2t^2}{m_0^2c^2}\right)^{1/2}}\left(\frac{Ft}{m_0c} + j1\right) \tag{3.11}$$

and from this

$$E_m\Big|_{t=0} = jm_0c^2 \tag{3.12}$$

i.e. as expected, the rest mass value, and is all temporal, [as for the same limit for (3.8)].
Re-arranging (3.11) to

$$E_m = \frac{m_0c^2}{\left(1+\dfrac{m_0^2c^2}{F^2t^2}\right)^{1/2}}\left(1+j\frac{m_0c}{Ft}\right) \tag{3.13}$$

shows that

$$E_m\Big|_{t\to\infty} = m_0c^2 \tag{3.14}$$

i.e. all spatial. However, this particular result is a hypothetical point in view of Section 3.2.3 below.

3.2.3. **Kinetic Energy.**

From (2.4) and (3.4) kinetic energy in spatial - temporal form is, after insertion of (3.4), given by

$$E_k = \left(Ftc + jm_0c^2\right)\frac{\left\{\left(1+\dfrac{F^2t^2}{m_0^2c^2}\right)^{1/2} - 1\right\}}{\left(1+\dfrac{F^2t^2}{m_0^2c^2}\right)^{1/2}} \tag{3.15}$$

and clearly

$$\begin{aligned} E_k\Big|_{t=0} &= 0 + j0 \\ E_k\Big|_{t\to\infty} &= \infty + jm_0c^2 \end{aligned} \tag{3.16}$$

and when $t \to \infty$ (3.16) confirms the expected result that it is the spatial kinetic energy that is the limiting factor in accelerated motion. Also, interestingly, the temporal component of kinetic energy when $t \to \infty$ assumes the same magnitude as that of the matter energy when $t = 0$.

3.3 Mass.

From [1] Eq.(3.6) the energy mass of the accelerated mass is simply

$$m = \frac{m_0}{\left(1-\dfrac{v^2}{c^2}\right)^{1/2}} \tag{3.17}$$

Insertion of (3.4) then gives

$$m = m_0\left(1+\frac{F^2t^2}{m_0^2c^2}\right)^{\frac{1}{2}} \tag{3.18}$$

and the limits with regard to t are obvious. Taking the time derivative of (3.18) however yields

$$\frac{dm}{dt} = \frac{F^2t}{m_0c^2\left(1+\frac{F^2t^2}{m_0^2c^2}\right)^{\frac{1}{2}}} \tag{3.19}$$

and the interesting limit here is

$$\left.\frac{dm}{dt}\right|_{t\to\infty} = \frac{F}{c} \tag{3.20}$$

i.e. Non-zero and finite.
The equivalent expression to (3.18) for inertial mass is simply

$$m_a = m_0\left(1+\frac{F^2t^2}{m_0^2c^2}\right)^{\frac{3}{2}} \tag{3.21}$$

and again the limits are obvious.

3.4 The Wavelength and Frequency of the Matter Wave.

In the text above, it was stated that the energy projected into the spatial domain exists in the form of a de Broglie matter wave, and in [2] it was also stated that the manner in which kinetic energy was stored by an accelerated mass was by an increase in its matter wave frequency. To express this in terms of time, consider [2], Eq.(2.18), repeated here for convenience.

$$\lambda_{sv} = \hbar\frac{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}{m_0v} \tag{3.22}$$

This is the spatial component of the matter wave wavelength of a particle mass with spatial velocity v and was stated in [2] to be the relativistic version of Louis de Broglie's quantum momentum hypothesis.
Inserting (3.4) for v, (3.22) becomes simply

$$\lambda_{sv} = \frac{\hbar}{Ft} \tag{3.23}$$

i.e. again Louis de Broglie' quantum hypothesis with $p = Ft$.
And consequently

$$f_{sv} = \frac{c}{\lambda_{sv}} = \frac{Fct}{\hbar} \tag{3.24}$$

and thus the frequency of the matter wave becomes a simple linear function of time, as would be expected from (3.8). (Note that this assumes that the velocity of propogation of the matter wave is equal to the speed of light).
Of course this relationship is only valid during the application of the accelerating force F. Once this is removed, determination of the matter wave wavelength can be derived from (3.22) with knowledge of the rest mass and the final velocity. Alternatively, (3.23) can be used if F and the precise time for which it was applied is known.

4.0 Conclusions.

The intent of this short paper was to show that the spatial-temporal projected distribution of energy, exhibited by quantum matter in motion, as demonstrated in [2], is also applicable to the motion of ponderable matter. This intent has been achieved and the components of this distribution have been shown to include both matter and kinetic complements. The most significant points to arise are that firstly, the spatial projection must therefore contain an increasing amount of matter energy as the spatial velocity increases. It is believed that this spatial component, E_s, including the matter energy component, as stated in the text, exists in the form of a Louis de Broglie matter wave. A means of determining the precise nature of this in detail however, including the mechanism that produces it, does not at the present seem clear. Secondly, it is also believed that the results show that the concept of negative energy, as apparently mathematically permitted via the square root of Einstein's energy-momentum relationship, is non-existent as a physical result.

It could be said that the assumption that the velocity of propogation of the matter wave being equal to the speed of light, is unjustified. For instance, in comparison to the velocity of propogation of water waves, or the standing waves of vibrating strings. However, the latter is the velocity of propogation of the undulations of real physical matter, rather than 'matter waves'. It is believed that the velocity of propogation of all such waves, i.e. electromagnetic and 'matter waves', is always at the velocity of light, albeit that this cannot, at the moment, be proved.

Apart from the above, the results here are largely of academic interest only. Expressing them as explicit functions of the time, has merely provided confirmation of limiting values that have already been determined from other means elsewhere.

References.

[1] P.G.Bass, *The Special Theory of Relativity - A Classical Approach.*, www.relativitydomains.com.

[2] P.G.Bass, *An Investigation of the Characteristics of de Broglie Matter Waves in the Relativistic Space-Time* $\boldsymbol{D_0}$, www.relativitydomains.com.