DERIVATION OF THE SCHRODINGER, KLEIN-GORDON AND DIRAC EQUATIONS OF PARTICLE PHYSICS VIA CLASSICAL METHODS.

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Abstract.

This paper presents the derivation of the Schrodinger, Klein-Gordon and Dirac equations of particle physics, for free particles, using classical methods. The derivations are based on the assumption that these wave equations are homogeneous and soluble via separation of variables. Several anomalies are highlighted and resolutions proposed. The analysis method employed is that of Relativistic Domain theory.

CONTENTS

1.0 Introduction.

In particle physics, particularly quantum mechanics and quantum electrodynamics, three equations that are of significant importance, and play a central role in the development of these subjects, are the Schrodinger, Klein-Gordon and Dirac equations. Derived in the early part of the 20th Century, they describe the wave like motion of material sub-atomic particles. The Schrodinger equation applies to particles in motion at 'non-relativistic' speeds, while the Klein-Gordon and Dirac equations represent particles in motion at velocities that are relativistically significant.

In the literature, it is frequently stated that these equations cannot be derived from first principles, and are effected via the replacement of particle momentum and energy with quantum operators. Accordingly, their existence, in lacking a formal derivation, is only justified on the basis that their solutions are successful in describing experimentally observed phenomena.
Here a classical derivation is effected for a free particle with a constant velocity. The derivation is not from first principles, but via the assumption that the solution for all homogeneous wave equations, is of the exact same form as Maxwell's wave equation of electrodynamics. This assumption is not without precedence as it appears frequently in the literature.

In this paper, the Schrodinger and Klein-Gordon equations are shown to possess anomalies with regard to their spatial wave function propagation velocities. These anomalies are easily resolved via the derivation method used, Relativistic Domain theory.
The initial part of the derivation of the standard Dirac equation, is a re-formulation of the Klein-Gordon, which is then augmented via the insertion of Dirac's gamma matrices, to account for both clockwise and anti-clockwise spin, and for both positive and negative energy solutions. It is only the initial part of the derivation that will be addressed here, followed by a re-interpretation of the second part.

For completeness, Maxwell's wave equation is derived and solved in Appendix A. Appendix B presents a further dissertation on the interpretation of Dirac's negative energy solution, while Appendix C provides an alternative formulation of the Klein-Gordon. Finally, Appendix D provides a brief review of the nature of wave propagation in the space-time of Relativistic Domains.

Note that a parameter, unless necessary for absolute clarity, will not be defined in this paper if it has already been so in references [1], [2], [3] and [5], with which familiarity is assumed.

2.0 Derivation of the Schrodinger Equation.

2.1 The Standard Equation.

As proposed in the Introduction and Appendix A, the solution to the wave function for a free particle possessing mass is

$$\Psi_x = \Psi_{x0} EXP\left\{\frac{i}{\hbar}\left(p_x x - Et\right)\right\} \tag{2.1}$$

Where Ψ_x is the wave function representing the material particle, and where, (as in Appendix A), only the x axis is considered for simplicity, and therefore p_x is the particle spatial momentum in that direction, and E is the particle's energy. For such a particle, in motion at a 'non-relativistic' velocity, in the standard literature

$$\begin{aligned} p_x &= m_0 v \\ E &= \frac{m_0 v^2}{2} \end{aligned} \tag{2.2}$$

So that (2.1) becomes

$$\Psi_x = \Psi_{x0} EXP\left\{ \frac{i}{\hbar}\left(m_0 vx - \frac{m_0 v^2}{2} t \right) \right\} \tag{2.3}$$

This when differentiated gives

$$\frac{\partial \Psi_x}{\partial t} = -\frac{i}{\hbar}\frac{m_0 v^2}{2}\Psi_x$$

and

(2.4)

$$\frac{\partial^2 \Psi_x}{\partial x^2} = -\frac{m_0^2 v^2}{\hbar^2}\Psi_x$$

which when combined yields

$$\frac{\partial^2 \Psi_x}{\partial x^2} + \frac{2im_0}{\hbar}\frac{\partial \Psi_x}{\partial t} = 0 \tag{2.5}$$

This is Schrodinger's equation of quantum mechanics for a free particle and is used extensively in much of the literature on the subject. However, this equation contains an anomaly regarding the wave function propagation velocity. Employing the Planck and de Broglie hypotheses

$$E = \frac{m_0 v^2}{2} = hf \ and \ p_x = m_0 v = \frac{h}{\lambda} \tag{2.6}$$

so that the propagation velocity is

$$\lambda f = \frac{m_0 v^2}{2h} \cdot \frac{h}{m_0 v} = \frac{v}{2} \tag{2.7}$$

Thus inferring that the wave propagation velocity is half that of the particle it is meant to represent. This is of course recognised in the literature, [6], and solved to show that it is the propagation velocity of the wave packet, (the group velocity), which is equal to the velocity of the material particle, and is therefore, accordingly twice that of the wave function, (the phase velocity). However, this is still not considered a valid result for such a particle wave motion. While it would be expected that the group velocity of the wave packet would be the same as that of the material particle, it would also be expected that, in line with the solution to Maxwell's wave equation, the phase velocity of the wave function would also be equal to the velocity of light.

The problem here lies in the value selected for the spatial momentum of the particle. This term lies along the spatial axis and is not coincident with the energy term, which does not lie along the *x* axis but along the Existence Velocity Vector, as shown in [1] and [2]. To address this it is first noted that there is no such motion that is 'non-relativistic'. All motion is relativistic irrespective of the smallness of the spatial velocity of the particle or object. Consequently, any treatment that assumes otherwise is an approximation. Therefore, all analysis of the type conducted here should be carried out along the

Existence Velocity Vector, because that is the true spatial-temporal path of the particle. Approximations for spatial velocities << *c* should then be determined from parameters that exist on this path. This is effected as follows.

2.2 The Revised Equation.

Derivation of the Schrodinger equation along the path of the Existence Velocity Vector is as follows. First the required energy term.

$$E_s = mc^2 = m_0 c^2 \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{2.8}$$

and when $v << c$

$$E_s = m_0 c^2 \tag{2.9}$$

For the momentum

$$p_s = mc = m_0 c \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{2.10}$$

So that when $v << c$

$$p_s = m_0 c \tag{2.11}$$

Insertion of (2.9) and (2.11) into (2.1) then gives

$$\Psi_s = \Psi_{s0} EXP\left\{\frac{i}{\hbar}\left(m_0 cs - m_0 c^2 t\right)\right\} \tag{2.12}$$

So that

$$\frac{\partial \Psi_s}{\partial t} = -\frac{i}{\hbar} m_0 c^2 \Psi_s$$

and

(2.13)

$$\frac{\partial^2 \Psi_s}{\partial s^2} = -\frac{m_0^2 c^2}{\hbar^2} \Psi_s$$

and consequently the combination of the two equations in (2.13) gives

$$\frac{\partial^2 \Psi_s}{\partial s^2} + \frac{i m_0}{\hbar} \frac{\partial \Psi_s}{\partial t} = 0 \tag{2.14}$$

Where the coefficient of the time derivative is now half that of the original equation. [1]. Taking the second time derivative in (2.13) gives

$$\frac{\partial^2 \Psi_s}{\partial t^2} = -\frac{m_0^2 c^4}{\hbar^2}\Psi_s \tag{2.15}$$

to give

$$\frac{\partial^2 \Psi_s}{\partial s^2} + \frac{1}{c^2}\frac{\partial^2 \Psi_s}{\partial t^2} = 0 \tag{2.16}$$

as per Maxwell. Consequently, it is easy to see from (2.9) and (2.11), that using the Planck and de Broglie hypotheses, the phase velocity is given by

$$\lambda_s f_s = c \tag{2.17}$$

This is the phase velocity of the wave function along the Existence Velocity Vector and is equal to the velocity of light. It is also the group velocity of the wave packet that contains the wave function along this vector. If θ is the spatial-temporal angle of the Existence Velocity Vector to the temporal axis, (see [1] and/or [2] and/or Appendix D), then the group velocity of the wave packet along the spatial axis is

$$\lambda_x f_x = c\,\mathbf{sin}\,\theta = c.\frac{v}{c} = v \tag{2.18}$$

i.e. the spatial velocity of the material particle itself along the same axis.
The phase velocity of the wave function, (within the wave packet), along the spatial axis, is determined via the projection of the particle's momentum and energy onto the spatial axis from the Existence Velocity Vector, (see [2] and /or Appendix D). The projection of energy onto the spatial axis is from (2.8)

$$E_x = E_s\frac{v}{c} = mc^2.\frac{v}{c} = mvc = m_0 vc\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{2.19}$$

where m is the energy mass, $m_0(1 - v^2/c^2)^{-1/2}$ (see [1]). The 'non-relativistic' approximation of (2.19) when $v << c$ is

$$E_x = m_0 vc \tag{2.20}$$

and for the particle momentum the projection is

$$p_x = p_s\frac{v}{c} = mc.\frac{v}{c} = mv = m_0 v\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{2.21}$$

and when $v << c$

$$p_x = m_0 v \tag{2.22}$$

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[1] *Note that retaining all terms in the expansion of E_s and p_s above in which the denominators are << c^2 makes no difference to the above result.*

From (2.20) and (2.22) the 'non-relativistic' projection of the wave function onto the x axis is therefore

$$\Psi_x = \Psi_{x0} EXP\left\{\frac{i}{\hbar}\left(m_0 vx - m_0 vct\right)\right\} \tag{2.23}$$

which leads to

$$\frac{\partial^2 \Psi_x}{\partial x^2} + \frac{1}{c^2}\frac{\partial^2 \Psi_x}{\partial t^2} = 0 \tag{2.24}$$

again as per Maxwell and with a phase velocity of

$$\lambda_x f_x = c \tag{2.25}$$

Expressed in the form of Schrodinger's original equation (2.24) is

$$\frac{\partial^2 \Psi_x}{\partial x^2} + \frac{im_0}{\hbar}\frac{v}{c}\frac{\partial \Psi_x}{\partial t} = 0 \tag{2.26}$$

and note that the coefficient of the time derivative is **sin**θ, *(v/c*) times that of (2.14).

Finally, note that reduced formulations of Schrodinger's equations derived here are obtained by the following substitutions

(i) Eq.(2.4) for $\frac{\partial \Psi_x}{\partial t}$ in (2.5), (spatial axis path).

(ii) Eq.(2.13) for $\frac{\partial \Psi_s}{\partial t}$ in (2.14), (Existence Velocity Vector path).

(ii) A similar substitution in (2.25), (spatial axis path, revised equation)

These all lead to

$$\frac{\partial^2 \Psi_x}{\partial x^2} + \frac{p^2}{\hbar^2}\Psi_x = 0 \tag{2.26}$$

For substitutions (i) and (iii), $p = m_0 v$ while for substitution (ii) $p = m_0 c$, (and read s for x).

All of the above results are discussed in detail in Section 6.0.

3.0 Derivation of the Klein-Gordon Equation.

3.1 The Standard Equation.

The Klein-Gordon equation is meant to represent a material particle in motion with a velocity of relativistic proportions. Its energy and momentum, as per the standard literature, are therefore given by

$$E = mc^2$$
$$p_x = mv \tag{3.1}$$

This provides a wave function of

$$\Psi_x = \Psi_{x0} EXP\left\{\frac{i}{\hbar}\left(mvx - mc^2 t\right)\right\} \tag{3.2}$$

So that

$$\frac{\partial^2 \Psi_x}{\partial t^2} = -\frac{m^2 c^4}{\hbar^2}\Psi_x$$

and (3.3)

$$\frac{\partial^2 \Psi_x}{\partial x^2} = -\frac{m^2 v^2}{\hbar^2}\Psi_x$$

and thus

$$\frac{\partial^2 \Psi_x}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \Psi_x}{\partial t^2} = \frac{m^2 c^2}{\hbar^2}\Psi_x - \frac{m^2 v^2}{\hbar^2}\Psi_x \tag{3.4}$$

which reduces to

$$\frac{\partial^2 \Psi_x}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \Psi_x}{\partial t^2} - \frac{m_0^2 c^2}{\hbar^2}\Psi_x = 0 \tag{3.5}$$

This is the standard Klein-Gordon equation of quantum electrodynamics, and like the standard Schrodinger equation possesses an anomaly with regard to its propagation velocity. This is determined as follows

$$E = mc^2 = hf \quad and \quad p_x = mv = \frac{h}{\lambda} \tag{3.6}$$

so that

$$\lambda f = \frac{mc^2}{h} \cdot \frac{h}{mv} = \frac{c^2}{v} \tag{3.7}$$

This is the standard phase velocity in the literature and which was shown in [3] to be fictitious because the f and λ there were non-coincident. The problem here is the same because the p_x and E in (3.6) are non-coincident. The p_x is the relativistic momentum on the x axis and the E is the total energy on the Existence Velocity Vector as depicted in [2].

In accordance with the commentary on the Schrodinger equation in the last Section, the analysis should be carried out on the Existence Velocity Vector and so the value of momentum required here is

$$p_s = mc \tag{3.8}$$

3.2 The Revised Equation.

Using the energy of (3.1) and the momentum of (3.8), the revised wave function becomes

$$\Psi_s = \Psi_{s0} EXP\left\{\frac{i}{\hbar}\left(mcs - mc^2 t\right)\right\} \tag{3.9}$$

and thus

$$\frac{\partial^2 \Psi_s}{\partial t^2} = -\frac{m^2 c^4}{\hbar^2}\Psi_s$$

and (3.10)

$$\frac{\partial^2 \Psi_s}{\partial s^2} = -\frac{m^2 c^2}{\hbar^2}\Psi_s$$

Which clearly shows that

$$\frac{\partial^2 \Psi_s}{\partial s^2} + \frac{1}{c^2}\frac{\partial^2 \Psi_s}{\partial t^2} = 0 \tag{3.11}$$

again as per Maxwell, and that

$$\lambda_s f_s = \frac{mc^2}{h} \cdot \frac{h}{mc} = c \tag{3.12}$$

Eq.(3.12) is both the group and phase velocities along the Existence Velocity Vector, and which, via multiplication by **sin**θ = *v/c*, ensures that the group velocity along the spatial axis is the same as that of the material particle along the same axis at the value *v*.
Via an identical process to the Schrodinger above, the phase velocity of the wave function within the wave packet on the spatial axis, can also be shown to be equal to the speed of light.
Finally, again as in the case of the Schrodinger, alternative formulations of the equation are available by appropriate substitutions as per (i), (ii) and (iii) in the previous Section. They all lead to

$$\frac{\partial^2 \Psi_x}{\partial x^2} + \frac{p^2}{\hbar^2}\Psi_x = 0 \tag{3.13}$$

where *p* in (i) and (iii) is *mv*, and in (ii) is *mc*, (and read *s* for *x*).

All of the above results are discussed in detail in Section 6.0.

4.0 The Stationary Particle.

If in the Klein-Gordon equations above, the spatial velocity *v*, is put to zero to produce a stationary particle, the following results are obtained. Note that in this case the temporal axis is also the Existence Velocity Vector. First for the original Klein-Gordon equation, the wave function becomes

$$\Psi = \Psi_0 EXP\left\{\frac{i}{\hbar}\left(-m_0c^2t\right)\right\} \tag{4.1}$$

Where Ψ is now a wave function purely on the temporal axis, so that

$$\frac{\partial^2\Psi}{\partial x_0^2} = 0$$

and (4.2)

$$\frac{\partial^2\Psi}{\partial t^2} = -\frac{m_0^2c^4}{\hbar^2}\Psi$$

to give

$$\frac{\partial^2\Psi}{\partial t^2} + \frac{m_0^2c^4}{\hbar^2}\Psi = 0 \tag{4.3}$$

then with

$$hf = m_0c^2 \quad and \quad \frac{h}{\lambda} = 0 \tag{4.4}$$

the phase velocity is

$$\lambda f = \infty \tag{4.5}$$

This suggests that the particle exists as a spatially stationary wave on the temporal axis with an infinite propagation velocity along it. This, in addition to (3.7), also contravenes Einstein's Special theory et al, and is therefore further reason to reject this construction of the original Klein-Gordon equation.

Performing the same exercise on the wave function for the revised Klein-Gordon equation, results in

$$\Psi = \Psi_0 EXP\left\{\frac{i}{\hbar}\left(m_0cx_0 - m_0c^2t\right)\right\} \tag{4.6}$$

which is identical to the revised Schrodinger equation along the Existence Velocity Vector, (2.12). Consequently it results in

$$\frac{\partial^2\Psi}{\partial x_0^2} + \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = 0 \tag{4.7}$$

again identical in form to Maxwell's equation and with a group/phase velocity of *c*.

Thus the spatially stationary wave propagates along the temporal axis with velocity *c*. However, because in Relativistic Domain theory, the temporal axis is in motion past the spatial axes with the same velocity, but in the opposite direction, (see [1]), the wave packet remains at the stationary location of the particle with a frequency of

$$f = \frac{m_0 c^2}{h} \tag{4.8}$$

Its wavelength on the temporal axis is consequently

$$\lambda = \frac{h}{m_0 c} \tag{4.9}$$

Putting $v = 0$ in the Schrodinger equations, original and revised, (spatial), eliminates the wave function completely, i.e. no spatial motion, as would be expected.

5.0 Derivation of the Dirac Equation from the Klein-Gordon Equation.

5.1 The Original and Revised Equations.

First, from the original Klein-Gordon, note that the first time derivative of (3.2) is

$$\frac{\partial \Psi_x}{\partial t} = -\frac{i}{\hbar} mc^2 \Psi_x \tag{5.1}$$

so that

$$\frac{\partial^2 \Psi_x}{\partial t^2} = \frac{1}{\Psi_x}\left(\frac{\partial \Psi_x}{\partial t}\right)^2 \tag{5.2}$$

Consequently (3.4) becomes

$$\frac{\partial^2 \Psi_x}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \Psi_x}{\partial t^2} - \frac{m_0^2 c^2}{\hbar^2}\Psi_x = -\frac{m^2 v^2}{\hbar^2} - \frac{1}{c^2 \Psi_x}\left(\frac{\partial \Psi_x}{\partial t}\right)^2 - \frac{m_0^2 c^2}{\hbar^2}\Psi_x = 0 \tag{5.3}$$

so that

$$-\frac{1}{c^2}\left(\frac{\partial \Psi_x}{\partial t}\right)^2 = \left(\frac{m^2 v^2}{\hbar^2} + \frac{m_0^2 c^2}{\hbar^2}\right)\Psi_x^2 \tag{5.4}$$

which becomes

$$-\hbar^2\left(\frac{\partial \Psi_x}{\partial t}\right)^2 = \left(p_x^2 + m_0 c^2\right)c^2 \Psi_x^2 \tag{5.5}$$

Also note that (5.5) is exactly equivalent to (5.1) so that this derivation is largely redundant. Its only purpose is to show that to this point the Dirac is indeed a simple variation of the Klein-Gordon.
Now expanding the momentum term in (5.5) to include the other two spatial axes, and therefore dropping the subscript *x* on Ψ, gives

$$-\hbar^2\left(\frac{\partial\Psi}{\partial t}\right)^2 = \left(p_x^2 + p_y^2 + p_z^2 + m_0c^2\right)c^2\Psi^2 \tag{5.6}$$

This is Dirac's equation prior to the insertion of the Dirac gamma matrices and subsequent factorisation. Using (3.2) however, means that it suffers from the same wave propagation anomaly as the original Klein-Gordon equation. Accordingly it should be derived from the wave function applicable to the Klein-Gordon equation along the Existence Velocity Vector. Performing this exercise however also results in (5.6). This coincidence occurs because the miscreant term in the derivation from the original Klein-Gordon, the spatial derivative of the spatial momentum, is eliminated via its incorporation in the constant term, to produce the total energy of the particle in the form of Einstein's energy/momentum relationship. In any case, as shown above, all that is in fact needed to obtain Dirac's equation to this point is (5.1), which avoids the momentum term in (3.2).
Nevertheless, however Dirac's equation is obtained to this point, for rigor, it is believed that derivation should start from the wave function of (3.9), to avoid the wave propagation anomaly associated with (3.2).

6.0 Discussion of Results.

6.1 The Schrodinger and Klein-Gordon Equations.

These two revised equations are discussed together because the latter is nothing more than the relativistic version of the former and vice-versa. For instance, if the revised Klein Gordon of (3.11) is expressed in the form of the Schrodinger equation, (2.14), it is

$$\frac{\partial^2\Psi_s}{\partial s^2} + \frac{im}{\hbar}\frac{\partial\Psi_s}{\partial t} = 0 \tag{6.1}$$

from which it can clearly be seen that the value of mass involved is the relativistic version of that in (2.14).

Also it should be noted that as with the comment associated with the stationary particle, both group and phase velocities are the same along the Existence Velocity Vector because along this vector all motion is at the velocity of light. This is a fundamental condition associated with this vector in Relativistic Domain theory, (see[1]). Relativistic Domain theory is fully derived in [1], concerning the motion of material objects, but it is perhaps necessary here to further explain the nature of wave propagation in that regime. This is considered a side issue and is therefore relegated to Appendix D.

With regard to the specific nature of the wave function, The Copenhagen Interpretation, largely due to Max Born, is that together with its complex conjugate, i.e. squared, it forms a probability density function, which is then integrated over all space to derive a probable location of the material particle. However, such an interpretation does not appear to have been applied to Maxwell's wave equation, in which the wave function is represented by the electric intensity, or alternatively the magnetic flux density, both possessing physical attributes. Consequently, in the Copenhagen Interpretation, it is not clear how a 'probability wave' would carry both the energy and momentum of the particle it is meant to represent, as is the case with the Maxwell. In addition, the precise relationship between the probability wave function, the wave packet and the material particle is unspecified.
It is consequently believed that an alternative to the Copenhagen Interpretation needs to be formulated. The nature of the wave function could be a proper wave contained within the wave packet in the same sense as an electromagnetic wave, but of a different unknown consistency. Hence the use of the term 'matter wave' in [3]. Alternatively, the wave function could simply be a physical

vibration of the material particle itself. However, it is not clear whether this scenario would produce the interference patterns exhibited in the double slit experiments. Also, such a particle vibration would require some form of driving force, and again it is not clear as to the source of such a force.
Consequently, of the above two possibilities, the former is preferred because of the precedent represented by the electromagnetic example.
With regard to the relationship between wave function and wave packet etc, it is believed that the wave function must be wholly contained within the wave packet, the envelope of which must be identical to the three dimensional envelope of the material particle. Outside of this envelope the wave function would be zero. Visually this would therefore be as in Fig. 6.1 below, (a slice through the middle of a spherical particle).

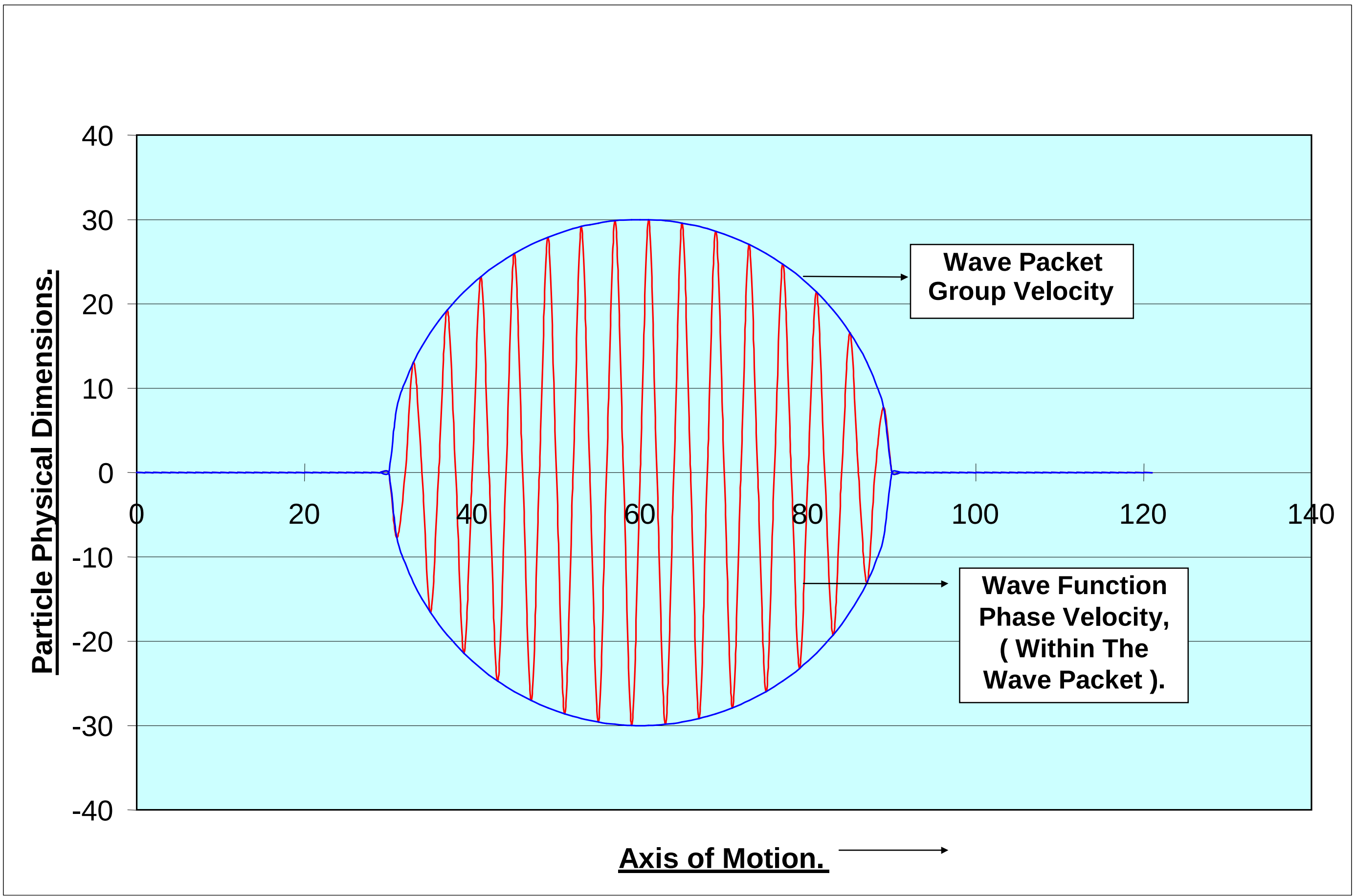

Fig. 6.1 - Conceptual Visualisation of a Particle Wave Function and Encompassing Wave Packet.

If the wave function were to extend outside of the envelope of the material particle, the energy and momentum distribution of each would not be identical, which must be considered an anomaly.
Finally, it is universally accepted that the wave function and the material particle cannot exist simultaneously. Therefore, either the particle exists only as a wave function/wave packet, as per (4.8) and (4.9), or, some mechanism would be required to effect the transformation between the wave and material existences, as a result of some trigger, possibly associated with its motion.

6.2 The Dirac Solutions.

6.2.1. Existing Interpretations.

Subsequent to the insertion of the Dirac matrices into (5.11) and factorisation, its solution provided a

description of particles with both positive and negative spin, and positive and negative energy. The positive energy solutions were then used to determine the nature of the wave function, (Dirac's Bi-Spinors), such as to represent the electron, (and all subsequent Fermion particles).
Dirac's interpretation of the negative energy solution was as a "hole" in a sea of negative energy electrons, which then appears as a positron, (with positive or negative spin). This interpretation is considered unlikely because it would require a multiple sea of negative energy charged particles to allow the definition of every anti-Fermion that exists.
An alternative interpretation, put forward by Richard P. Feynman, [4], is that a negative energy electron travelling backwards in time, is equivalent to a positive energy positron travelling forwards in time. While such an interpretation is more conducive to describing all anti-particles, it means that the negative sign attached to the energy, must first be transferred to the flow of time, and then to the charge. This is not only considered a dubious practice physically, but also mathematically. However, this interpretation is not universally accepted, as it is effectively "rejected" in [7].
Irrespective of this, in Relativistic Domain theory, matter cannot travel through time, it is the other way round, time flows past three dimensional space and therefore past all matter that exists within it. Matter cannot therefore travel backwards in time, (see [1], [3] and [5] and Appendix B), and this interpretation of the negative energy solution of Dirac's equation is consequently unacceptable.
Accordingly, it is necessary to provide new interpretations of Dirac's solutions to account for all Fermion anti-particles and for the negative energy result.

6.2.2. **The Positive Energy Solution Re-Interpretation.**

First for the anti-particles. Such particles should not be referred to as "anti-matter", because they are identical in every way to normal particles except for their electric charge, which is simply a reversal of sign. Now, the precise nature of electric charge is not known, i.e. it cannot be derived from first principles involving other fundamental parameters, and indeed may be a fundamental parameter itself. Accordingly, until this question is resolved, it is believed that electric charge should not be artificially introduced into any parametric relationship meant to represent fundamental particles such as electrons or positrons etc. Instead only the positive energy solution of Dirac's equation, with positive and negative spins, should be regarded as a complete solution for all particles and anti-particles, and electric charge, positive or negative, then "assigned" to that solution to produce the final result. While perhaps not such an elegant way of defining particles and anti-particles, it avoids the artificially constructed derivations discussed above.

6.2.3. **The Negative Energy Solution Re-Interpretation.**

The negative energy solutions, if such particles exist, should be regarded as true anti-matter for reasons discussed below. Firstly, such particles would, according to Einstein's energy/mass relationship, exhibit negative mass. As such they would produce a repulsive gravitational field and, accordingly not accumulate into the huge conglomerations of atoms exhibited by positive energy particles, i.e. all forms of ponderable matter including planets, stars and galaxies etc. Instead they would most probably be evenly distributed throughout inter-stellar and inter-galactic space. If possessing spin and/or charge, some small accretions may exist but probably only in the form of simple anti-matter atoms.
Also, they would be completely invisible in a positive energy environment. Any impact with for instance light, i.e. photons, would not result in any form of reaction such as reflections etc, the light bearing its positive energy would simply be absorbed by the anti-matter particle, whose negative energy would thereby be reduced by the applicable amount. Such a negative energy particle would then exist in an excited state, and may subsequently decay back to its ground state by emitting a positive energy particle, thus contributing to the range of cosmic radiation that abounds in all of

space. Accordingly, it is clear that such negative energy anti-matter particles could contribute to, or even fully represent, what is currently envisaged as "Dark Matter".
Finally, this interpretation may also explain why there appears to be a preponderance of positive energy matter over anti-matter. A preponderance which may therefore be only illusionary.

7.0 Conclusions.

The fact that the revised Schrodinger equation of (2.14), and the Klein-Gordon as expressed as (6.1), are identical apart from the value of the particle mass, is not an unexpected result, because the only difference here is the physical velocity of the material particle, and in the Klein-Gordon case, the effect that it has on its mass. In both cases the group velocity is shown to be identical to that of the material particle, while the respective phase velocities remain identical, and independent of spatial-temporal direction, a statement that includes the temporal axis.

The interpretation that the wave function is considered to be a "matter wave", the precise nature of which is unknown, in opposition to the statistical interpretation of the Copenhagen Agreement, will be considered somewhat controversial. However, if the combination wave packet/wave function is to properly represent the material particle, then it must be capable of representing all of the attributes possessed by that particle. Consequently the wave packet/wave function needs to carry not only the momentum and total energy, but also needs to be contained within the physical constraints of the particle, to ensure equilibrium of energy and momentum distribution. In addition, where the particle exhibited spin, so also should the wave packet spin, which would result in the in the path of the wave function within it being spiral.

The Dirac equation interpretations proposed here, are almost entirely the result of the space-time environment in which the analysis has been conducted, the Relativistic Domain regime. In this regime it has, via referenced analysis, been shown that travel through time is not possible. Consequently, this has resulted in a re-interpretation of both the positive and negative energy solutions. In the case of the positive energy solution, the re-interpretation emphasises the question concerning the precise nature of electric charge. Either this is an attribute that can be derived from more basic parameters, in which case it could be properly built into a re-derivation of the Dirac gamma matrices/bi-spinors. Alternatively, if charge is to be confirmed as a basic attribute of nature, then in derivations such as is the concern here, it would have to be an assigned attribute as recommended at the moment in this paper.
For the negative energy solution, if negative energy particles exist, it will become necessary to determine some means of detection and characterisation. Would it be a single elementary particle or, as is the case with their positive energy counterparts, exist as a family of particles to form a range of anti-matter "quarks" et al. Until such deliberations have been effected, the existence of Dark Matter may remain uncertain, and whether such anti-matter particles contribute to or even fully represent it.

Appendix A.

The Maxwell Wave Equation and Solutions.

In this Appendix only, E is electric intensity, and energy is represented by ε

A.1 Derivation.

Maxwell's equations of electromagnetics are

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla . E = 0$$
$$\nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t} \qquad \nabla . B = 0 \tag{A.1}$$

Maxwell's wave equation of electromagnetic transmission is then simply derived as follows. From (A.1)

$$\begin{aligned}\nabla \times \nabla \times E &= -\frac{\partial}{\partial t}(\nabla \times B) \\ &= -\frac{\partial}{\partial t}\left(\frac{1}{c^2}\frac{\partial E}{\partial t}\right) \\ &= -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}\end{aligned} \tag{A.2}$$

From vector theory and (A.1)

$$\nabla \times \nabla \times E = \nabla(\nabla . E) - \nabla^2 E = -\nabla^2 E \tag{A.3}$$

So that the combination of (A.3) with the last term in (A.2) gives

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \tag{A.4}$$

This is Maxwell's wave equation for electric intensity in free space. A similar process yields

$$\nabla^2 B - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} = 0 \tag{A.5}$$

as the wave equation for magnetic flux density in free space.

A.2 Solution as a Wave.

Considering just the *x* axis for simplicity, (A.4) becomes

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \tag{A.6}$$

In concert with normal practice, this type of homogeneous equation can be solved via separation of variables and has a solution, (where the constant of integration is, $E(x,t) = E_0$ when $x = t = 0$), of

$$E(x,t) = E_0 EXP[i(kx - \omega t)] \tag{A.7}$$

where

k = spatial frequency = $2\pi/\lambda$

ω = temporal frequency = $2\pi f$

Substituting (A.7) into (A.6) yields

$$\frac{\partial^2 E}{\partial x^2} = -k^2 E$$

and (A.8)

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E$$

So that (A.6) becomes

$$\left(-k^2 + \frac{\omega^2}{c^2}\right)E = 0 \tag{A.9}$$

and therefore

$$k = \frac{\omega}{c} \tag{A.10}$$

to give finally

$$\lambda f = c \tag{A.11}$$

as the velocity of propagation.

A.3 Solution as a Particle.

By the Planck hypothesis, the energy of a particle, in this case a photon, is

$$hf = \hbar\omega = \varepsilon \tag{A.12}$$

and by the de Broglie hypothesis, the momentum of a particle is

$$\frac{h}{\lambda} = \hbar k = p \tag{A.13}$$

Then (A.7) becomes,

$$E(x,t) = E_0 EXP\left[\frac{i}{\hbar}(px - \varepsilon t)\right] \tag{A.14}$$

Via a process identical to that above

$$\frac{\partial^2 E}{\partial x^2} = -\frac{p^2}{\hbar^2} E$$

and (A.15)

$$\frac{\partial^2 E}{\partial t^2} = -\frac{\varepsilon^2}{\hbar^2} E$$

So that (A.6) becomes

$$\left(-\frac{p^2}{\hbar^2} + \frac{\varepsilon^2}{c^2\hbar^2}\right)E = 0 \tag{A.16}$$

which finally gives

$$\varepsilon = pc \tag{A.17}$$

So that all of the energy in the photon is kinetic energy due to its motion at the speed of light. This infers that the mass of the photon is zero. Essentially here, the photon is the wave packet that carries the wave function, the electric intensity *E*, along the *x* axis which, in this case, also represents the Existence Velocity Vector, so that both the group velocity of the wave packet and the phase velocity of the wave function are the same at the speed of light.
It is proposed that the solution for the particle obtained here, is applicable to all homogeneous wave equations, including those representing particles possessing mass.

Appendix B.

Additional Comments on Dirac's Negative Energy Solution.

In this Appendix, the source of the negativity in Dirac's negative energy solution is discussed. Dirac's negative energy solution stems ostensibly from Einstein's energy/momentum relationship, repeated here for convenience.

$$E = \left(p^2c^2 + m_0^2c^4\right)^{1/2} \tag{B.1}$$

possessing both positive and negative roots. Eq.(B.1) can easily be converted to Einstein's energy/mass relationship thus. From (B.1)

$$E = \left[m_0^2 c^4 + \frac{m_0^2 c^2 v^2}{\left(1 - \frac{v^2}{c^2}\right)} \right]^{1/2} \tag{B.2}$$

$$= \frac{m_0 c^2}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \left[1 - \frac{v^2}{c^2} + \frac{v^2}{c^2}\right]^{1/2} \tag{B.3}$$

$$= \frac{m_0 c^2}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} (1)^{1/2} \qquad \text{(B.4)}$$

Because, (in Relativistic Domain Theory), matter cannot flow through time, the term $\left(1 - v^2/c^2\right)^{1/2}$ must be positive. In any case, because (B.4) applies whatever the value of the spatial velocity v, putting $v = 0$ gives

$$E = m_0 c^2 \frac{(1)^{1/2}}{(1)^{1/2}} = m_0 c^2 \qquad \text{(B.5)}$$

Consequently, the only way for the energy E to be negative, is for the rest mass m_0 to be negative. This would require some mechanism to exist capable of creating negative mass. If the Higgs mechanism is one that creates mass, it is not clear how it could produce a negative mass. Therefore this means that some other separate mechanism would be necessary.
On the other hand, the above is predicated on the assumption that it is mass that is the fundamental parameter, and energy is then a derivation of it as per (B.5). Alternatively, if it is energy that is the fundamental parameter, then mass would result from a simple variation of Einstein's equation, i.e.

$$m_0 = \frac{E}{c^2} \qquad \text{(B.6)}$$

This latter interpretation would refute the Higgs Mechanism and assert that it is energy that exists and/or is created, in both positive and negative forms.

Appendix C.

An Alternative Derivation of the Klein-Gordon Equation.

This Appendix presents an alternative derivation of the Klein-Gordon equation along the spatial axis, in which the spatial velocity v appears explicitly. This is effected by direct use of Einstein's energy/momentum relationship in the solution for the wave function. From (3.8) the energy projected into the spatial axis is

$$\begin{aligned} E_x &= \left(pc^2 + m_0^2 c^4\right)^{1/2} \frac{v}{c} \\ &= \left(m^2 v^4 + m_0^2 v^2 c^2\right)^{1/2} \end{aligned} \qquad \text{(C.1)}$$

and the projected momentum is

$$p_x = mc.\frac{v}{c} = mv \qquad \text{(C.2)}$$

So that the wave function becomes

$$\Psi_x = \Psi_{x0} Exp\left[\frac{i}{\hbar}\left\{mvx - \left(m^2v^4 + m_0^2v^2c^2\right)^{1/2} t\right\}\right] \tag{C.3}$$

Taking the partial derivatives

$$\frac{\partial^2\Psi_x}{\partial t^2} = -\left(m^2v^4 + m_0^2v^2c^2\right)\frac{\Psi_x}{\hbar^2}$$

and (C.4)

$$\frac{\partial^2\Psi_x}{\partial x^2} = -\frac{m^2v^2}{\hbar^2}\Psi_x$$

and this leads directly to

$$\frac{\partial^2\Psi_x}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2\Psi_x}{\partial t^2} - \frac{m_0^2c^2}{\hbar^2}\Psi_x = 0 \tag{C.5}$$

an equation identical to the original Klein-Gordon, apart from the coefficient of the time derivative. This coefficient appears to suggest that the velocity of propagation of the wave function is the same as that of the material particle on the spatial axis, but this is not so. The phase velocity is determined as before, with

$$f_x = \frac{\left(m^2v^4 + m_0^2v^2c^2\right)^{1/2}}{h}$$

and (C.6)

$$\lambda_x = \frac{h}{mv}$$

then

$$\begin{aligned} \lambda_x f_x &= \frac{\left(m^2v^4 + m_0^2v^2c^2\right)^{1/2}}{h} \cdot \frac{h}{mv} \\ &= \left(v^2 + \frac{m_0^2}{m^2}c^2\right)^{1/2} \\ &= c \end{aligned} \tag{C.7}$$

i.e. the same as in all other derivations. Note that the group velocity would still be equal to *v*.

APPENDIX D.

Wave Packet/Wave Function Propagation in Relativistic Domain Theory.

First a brief résumé. Consider the existence of a material object in the Space-Time represented by Relativistic Domain theory. This is briefly reviewed below via reference to Fig. D.1. For an object at A that is spatially stationary, due to the motion of the temporal axis in the direction shown, its Existence Velocity Vector is represented by the line A → B, with magnitude *c*, the speed of light.

If the object at A possesses a spatial velocity, its Existence Velocity Vector rotates by the appropriate angle θ in the spatial - temporal plane, to lie along the line A → C. Its magnitude is still *c*, but is now

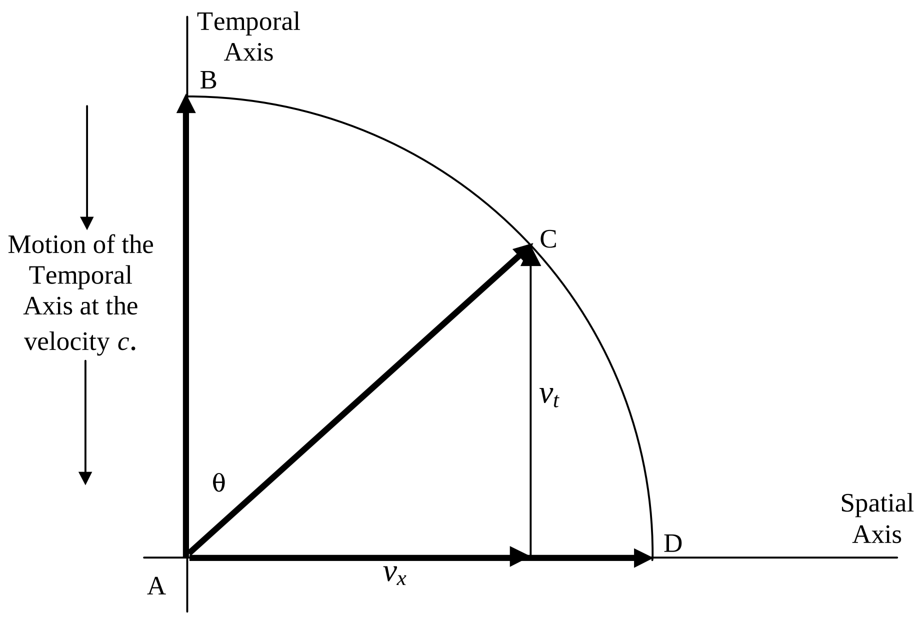

Fig. D.1 - Existence in the Relativistic Domain Space-Time.

made up of its spatial velocity

$$v_x = c\,\mathbf{sin}\,\theta \tag{D.1}$$

and its temporal velocity which is reduced to

$$v_t = c\,\mathbf{cos}\,\theta = c\left(1 - \frac{v_x^2}{c^2}\right)^{1/2} \tag{D.2}$$

This results in a reduction in the rate of passage of time for the object in motion and an increase in its mass.

If the object at A possesses a spatial velocity of *c*, then its Existence Velocity Vector lies along the line A → D and its temporal velocity is zero. Hence under this scenario the rate of passage of time for the object is zero. The only object capable of this existence is the photon which has zero rest mass.
A more detailed dissertation on the above is provided in [1].

A wave packet in such a regime can be treated in an identical manner to the object above, because it is a complete entity whose envelope is identical to the material particle it represents. The nature of the wave function within the wave packet of composite particles may well be the composite of the wave functions of the individual elementary particles that make it up. These wave functions would differ according to the different mass of each individual elementary particle.

With regard to the wave function itself, albeit that the nature of this parameter is unknown, it is clear that it cannot be treated in the same manner as the wave packet above. It is not the function, or its propagation velocity, that is dependent upon the spatial - temporal angle θ between the existence Velocity Vector and the temporal axis, but its frequency and wavelength. This was demonstrated in [3], and is also obvious from the treatment in this paper wherein it is both energy and momentum that are subject to this dependency, by virtue of the fact that these parameters are linked to frequency and wavelength via the Planck and de Broglie hypotheses. However, because frequency and wavelength are inversely proportional, the effects of the relationship with θ cancel, and the propagation velocity of the wave function remains the same along the spatial axis at the speed of light. It is clear that if this were not so, and the propagation velocity of the wave function decreased in line with the sine of θ, the interference patterns obtained with double slit particle experiments, would be vastly different to what has been found. Thus in order to derive the various wave equations, in the manner demonstrated, as they appear on the spatial axis, it is necessary to determine projected energy and momentum of the particle on that axis.

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