AN INVESTIGATION INTO THE CHARACTERISTICS OF DE BROGLIE MATTER WAVES IN THE RELATIVISTIC SPACE-TIME DOMAIN D_0, (PSEUDO-EUCLIDEAN SPACE-TIME).

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ABSTRACT.

This paper investigates the characteristics of de Broglie's matter waves in the Relativistic Space-Time Domain $\mathbf{D_0}$, (Pseudo-Euclidean Space-Time), and thereby provides a simplified theoretical demonstration of the dual nature of matter in both particulate and wave function form. These characteristics then enable a new method of derivation of Einstein's relativistic energy-momentum relationship which subsequently allows a deeper insight into the nature of the spatial-temporal distribution of the energy of matter.

CONTENTS.

1.0 Introduction.

During the early part of the 20th Century, Max Planck proposed that the emission and absorption of light takes place in packets or quanta. The relationship advanced was that the energy of the emitted/absorbed radiation, was related to its frequency via a simple constant of proportionality, subsequently termed Planck's constant. Einstein then extended this idea as inherent in the very nature of all propagating electromagnetic radiation. Subsequently, Louis de Broglie, (pronounced "de Broy"), went further and postulated that this concept applied to all matter of whatever nature. The hypothesis advanced by de Broglie, in concert with Planck's relationship, was that all matter possessed a "matter wave", such that its inverse wavelength was related to mechanical momentum via Planck's constant.

This paper investigates the characteristics of de Broglie's matter waves within Pseudo-Euclidean Space-Time, as represented by the Relativistic Space-Time Domain $\mathbf{D_0}$, (See [1]). It will thereby provide a simple theoretical demonstration that matter, when possessing a spatial velocity within such a Domain, exists simultaneously in dual form - with both a particulate and a wave function existence. This then enables a number of concepts with regard to the nature of matter energy to be investigated. It will also provide the means by which the resurrection of the Bohr/Sommerfeld old quantum theory of atomic structure can be effected, and to provide that theory with an equal or better theoretical basis, to that of the modern quantum mechanics/quantum electrodynamics theory. This particular concept will form the subject of a future series of papers.

This paper will include mathematical development in relativistic spatial/temporal form. Ref. [1] is particularly important in understanding such developments, and should therefore be read thoroughly first.

2.0 Matter Waves in the Relativistic Space-Time Domain $\mathbf{D_0}$, (Pseudo-Euclidean Space Time).

2.1 General Concepts.

In the particulate - matter wave dualism of existence of matter, the relationships proposed by Max Planck and Louis de Broglie were

$$E = hf \quad \text{and} \quad M = h\varpi \tag{2.1}$$

where

- E is the total energy of the matter wave
- h is Planck's constant of proportionality, (≈ 6.62E-27 erg secs).
- f is the frequency of the matter wave.
- M is the momentum of the matter wave particle.
- ϖ is the wave number, (reciprocal of wavelength), of the matter wave.

In Relativistic Domain theory, the parallel relationships are, [1]

$$E = mc^2 = \frac{m_0 c^2}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

and (2.2)

$$\mathrm{M} = m\left\{\boldsymbol{v} + \boldsymbol{j}c\left(1 - \frac{v^2}{c^2}\right)^{1/2}\right\}$$

where

- $\mathbf{M}$ is the spatial-temporal momentum of the particle, (see [1]).
- m is the energy mass of the particle.
- m_0 is the rest mass of the particle.
- $\boldsymbol{v}$ is the spatial vector velocity of the particle.
- c is the spatial terminal velocity in $\mathbf{D_0}$, (≈ the velocity of light), and is also the magnitude of Existence Velocity in $\mathbf{D_0}$, (see [1]).
- $\boldsymbol{j}$ is a unit vector in the temporal direction.

If the components of (2.1) and (2.2) are equated, it is immediately clear that the momentum component of (2.1) must be expanded to possess both a spatial and a temporal part thus

$$\mathrm{M} = h\left(\boldsymbol{\varpi}_s + \boldsymbol{j}\varpi_\tau\right) \tag{2.3}$$

Where

- $\boldsymbol{\varpi}_s$ is the wave number of a matter wave in a spatial vector direction.
- ϖ_t is the wave number of a matter wave in the temporal direction.

It will be seen later that this type of expanded representation will also be applicable to the energy component of (2.1).
The implications of the equivalence of (2.1), (with (2.3) inserted), and (2.2) are explored in the following three Subsections. Before the general case of a finite spatial velocity is considered, the two extreme cases of $|\boldsymbol{v}| = 0$ and $|\boldsymbol{v}| = c$ are briefly reviewed.

2.2 A Stationary Particle.

2.2.1 Energy.

Equating the energy components of (2.1) and (2.2) for a stationary particle, $|\boldsymbol{v}| = 0$ gives,

$$m_0 c^2 = h f_0 \tag{2.4}$$

where

f_0 is the matter wave frequency of a stationary particle.

2.2.2 Momentum.

Equating the momentum components of (2.2) and (2.3), noting that ϖ_s must be zero, gives

$$\boldsymbol{j}\, m_0 c = \boldsymbol{j}\, h\varpi_{t0} \tag{2.5}$$

where

ϖ_{t0} is the temporal wave number of a stationary particle.

Clearly from (2.4) and (2.5)

$$\boldsymbol{j}\, f_0 \boldsymbol{\lambda}_0 = \boldsymbol{j}\, c \tag{2.6}$$

This shows that for a stationary particle, the associated matter wave propagates along the temporal axis with the velocity c. Its existence within the spatial dimension is solely corpuscular with zero spatial velocity. Its existence within $\mathbf{D_0}$, in both forms therefore conforms to the criterion of existence within that Domain, (see [1]).

2.3 A Terminal Velocity Particle.

2.3.1 Energy.

Equating the energy components of (2.1) and (2.2) for such a particle, $|\boldsymbol{v}| = c$, yields

$$h f_c = \left. \frac{m_0 c^2}{\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}} \right|_{v \to c} \tag{2.7}$$

where

f_c is the frequency of the matter wave for a terminal velocity particle.

If the limit in (2.7) is explored the RHS becomes infinite, but a simple re-arrangement gives

$$m_0 = \frac{hf_c}{c^2}\left(1-\frac{v^2}{c^2}\right)^{1/2}\Bigg|_{v\to c} \tag{2.8}$$

and this shows that the only way for matter energy to achieve a velocity of *c*, the spatial terminal velocity of $\mathbf{D_0}$, is for it to possess zero rest mass. The energy of such a particle must therefore all be kinetic in nature. This suggests that if such a particle were brought to rest, all of its energy would be transferred to the arresting body and the particle itself cease to exist. For instance, consider the Compton effect. By the law of conservation of energy, from [2], Appendix X,

$$hf_c + m_{e0}c^2 = hf_c'' + m_e c^2 \tag{2.9}$$

where

f_c''	is the frequency of deflected light after the collision.
m_{e0}, m_e	is the mass of the electron before and after the collision.

If the collision is such that f_c'' is zero, i.e. a "head on" collision, then the mass of the electron after the collision is, from (2.9)

$$\begin{aligned} m_e &= m_{e0} + \frac{hf_c}{c^2} \\ &= \frac{m_{e0}}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \end{aligned} \tag{2.10}$$

Thus the energy given up in this exchange is as per (2.8). This energy is transferred to the electron as kinetic energy and the electron acquires a velocity of, from (2.10)

$$v = c\left\{1-\frac{1}{\left(1+\frac{hf_c}{m_{e0}c^2}\right)^2}\right\}^{1/2} \tag{2.11}$$

It will be shown later that this kinetic energy is transferred to the electron as an increase in its own matter wave frequency.

2.3.2 Momentum.

Equating the momentum components of (2.2) and (2.3) under this condition gives

$$h\varpi_c = \left.\frac{m_0 c}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}\right|_{v\to c} \qquad (2.12)$$

Where

ϖ_c is the spatial wave number of a particle when $|v| \to c$, (note that ϖ_t in (2.3) is zero in this case).

so that

$$\lambda_c = \left.\frac{h\left(1-\frac{v^2}{c^2}\right)^{1/2}}{m_0 c}\right|_{v\to c} \qquad (2.13)$$

and therefore from (2.7) and (2.13)

$$f_c \lambda_c = c \qquad (2.14)$$

The matter wave for such a particle therefore propagates entirely along the spatial axes at the terminal velocity in $\mathbf{D_0}$, i.e. c. For this particle, time is stationary and it possesses no temporal existence at all. Both f_c and λ_c are indeterminate from the above analysis and depend upon the characteristics of the emitting medium.

2.4 A Particle With a Finite Spatial Velocity.

This is the general case.

2.4.1. Energy.

First equating the energy components of (2.1) and (2.2) gives

$$hf_v = \frac{m_0 c^2}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \qquad (2.15)$$

and thus from (2.4)

$$f_v = \frac{f_0}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \qquad (2.16)$$

With reference to this result, it was stated in [1] that when a material body is spatially accelerated from rest, the kinetic energy it gains was stored as an increase in mass. From (2.16) it is now clear that the manner in which that storage takes place is by an increase in the matter wave frequency of the accelerated body as mentioned in relation to the Compton effect in Subsection 2.3.1. Consequently, it is also seen from (2.16) that to spatially accelerate any mass to the terminal velocity *c* would result in its matter wave frequency becoming infinite. A result also evident in (2.7). Therefore from the energy component of (2.1) this would require an infinite amount of accelerating energy, a result that concurs which other analyses in the literature, i.e.[1].

2.4.2 Momentum.

Now consider the momentum components of (2.1) and (2.2), noting that the full spatial-temporal form of (2.1) is now invoked, i.e. (2.3)

$$\begin{aligned} h\left(\boldsymbol{\varpi}_{sv} + j\varpi_{tv}\right) &= m\left\{\boldsymbol{v} + \boldsymbol{j}\left(1-\frac{v^2}{c^2}\right)^{1/2}\right\} \\ &= \frac{m_0\,\boldsymbol{v}}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} + \boldsymbol{j}\,m_0 c \end{aligned} \tag{2.17}$$

First consider the spatial components of (2.17), these give

$$\lambda_{sv} = \frac{h\left(1-\frac{v^2}{c^2}\right)^{1/2}}{m_0 v} \tag{2.18}$$

Where *v* is the magnitude of ***v***.

This is the relativistic version of de Broglie's equation which will be central in the resurrection of the Bohr/Sommerfeld theory of atomic structure in a future series of papers. Combining (2.15) and (2.18) gives

$$f_v\lambda_{sv} = \frac{c^2}{v} \tag{2.19}$$

This is the so called 'phase velocity' of the matter wave and in view of the fact that it is greater than the velocity of light, it has been stated in the literature, [2] to have no physical significance. Such a statement needs clarification which will be provided in the next Section.

Now consider the temporal parts of (2.17), they yield

$$\boldsymbol{j}\boldsymbol{\lambda}_{tv} = \boldsymbol{j}\frac{h}{m_0 c} \tag{2.20}$$

This result is identical to (2.5), (for a stationary particle), and confirms that the temporal component of the particle matter wave is unchanged by the spatial motion. This is because the motion has been spatially induced and no energy has been either added or subtracted in the temporal direction. However, from (2.15) and (2.20)

$$f_v\lambda_{tv} = \frac{c}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \tag{2.21}$$

This is the 'phase velocity' in the temporal direction and is again seen to be greater than the terminal velocity of $\mathbf{D_0}$. Eq(2.21) along with (2.19) will be discussed in detail in the next Section.

Finally consider the magnitude of (2.17), this yields

$$\lambda_v = \frac{h\left(1-\frac{v^2}{c^2}\right)^{1/2}}{m_0 c} \tag{2.22}$$

and therefore from (2.15)

$$f_v\lambda_v = c \tag{2.23}$$

Clearly, from (2.15), (2.17) and (2.23) the matter wave is propagating along the same path as the Existence Velocity Vector of the particle, (see [1]), and at the same velocity. In addition, it is now clear that this is also the case for the stationary particle in Subsection 2.1 and the terminal velocity particle in Subsection 2.2. This now helps the clarification in the next Section of the matter wave 'phase velocities' of (2.19) and (2.21).

2.5 Matter Wave Phase Velocities.

These are purportedly the velocities with which the matter wave propagates through the spatial dimension, (2.19), and the temporal dimension, (2.21). Both of these velocities are greater than the terminal velocity in $\mathbf{D_0}$, and so contravene both the primary criterion of existence in that Domain and Einstein's maximum velocity of spatial propagation. This apparent conflict has arisen because the product of the parameters in (2.19) and (2.21) are in fact invalid, for the following reason. The matter wave frequency f_v is the frequency of the matter wave as it propagates along the same path as the Existence Velocity Vector of the associated particle. The wavelength λ_{sv} is that of the spatial component of this matter wave, and the wavelength λ_{tv} is that of the temporal component. These frequency and wavelength parameters are not coincident and therefore the products of (2.19) and (2.21) are invalid as true velocities. To resolve this it is necessary to associate with λ_{sv} in the spatial dimension a frequency such that

$$f_{sv}\lambda_{sv} = c \tag{2.24}$$

and with λ_{tv} in the temporal dimension a frequency such that

$$f_{tv}\lambda_{tv} = c \tag{2.25}$$

It is then clear from (2.19) and (2.21) that

$$f_{sv} = f_v \frac{v}{c}$$
$$f_{tv} = f_v \left(1 - \frac{v^2}{c^2}\right)^{1/2} \qquad (2.26)$$

and it is also clear that f_{sv} and f_{tv} are the projection of f_v into the spatial and temporal dimensions respectively.
As a result of this it is now possible to represent the dual corpuscular/wave function existence of matter in $\mathbf{D_0}$ as in Fig. 2.1 below.

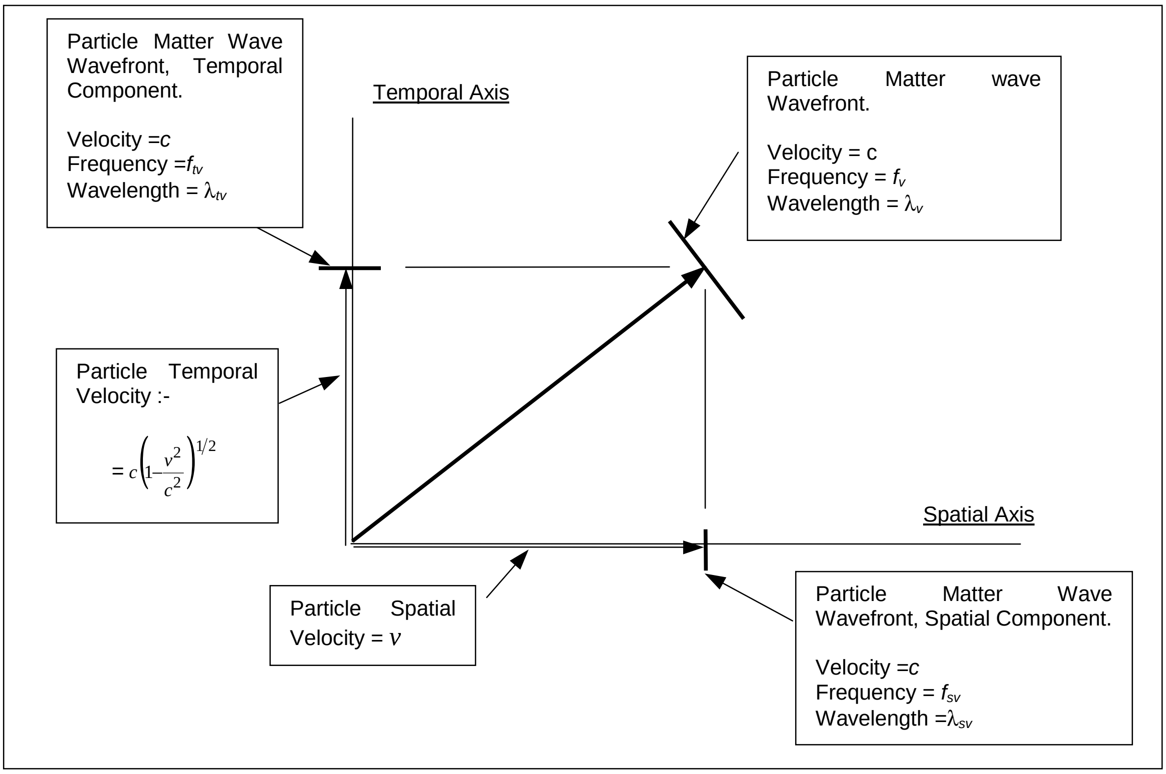

Fig. 2.1 Pictorial Representation of the Matter Wave Associated with a Particle in $\mathbf{D_0}$

It is emphasised that Fig. 2.1 is not a literal representation, only a pictorial one.

Note that the associations of (2.24), (2.25) and (2.26) now require that energy as well as momentum has both spatial and temporal components. This is also evident from (2.2) and (2.6), and is further analysed in the next Section

From (2.26) it is a simple matter to determine the relationship between the wavelength components of the wave.

3.0 The Nature of Matter Energy in the Relativistic Space-Time Domain $\mathbf{D_0}$.

3.1 Einstein's Relativistic Energy/Momentum Relationship.

Via a simple re-arrangement of (2.1) using (2.23), this relationship can be derived easily as follows. Here $\mathbf{E}$ is assumed to possess both spatial and temporal components as advanced earlier. Accordingly, its terms have been so designated in the following derivation.

$$\mathrm{E} = h\mathrm{f}_{\mathrm{v}} = \frac{hc}{\boldsymbol{\lambda}_{\mathrm{v}}} = h\boldsymbol{\varpi}_{\mathrm{v}}c = \mathrm{M}c \tag{3.1}$$

Where

$\mathbf{E}$ is spatial - temporal total energy.
$\mathbf{f_v}$ is spatial - temporal matter wave frequency.
$\boldsymbol{\lambda_v}$ is spatial - temporal matter wave wavelength.
$\boldsymbol{\varpi_v}$ is spatial - temporal matter wave wave number.

and thus from (2.2) and (3.1)

$$\begin{aligned}\mathrm{E} &= mc\left\{\boldsymbol{v} + \boldsymbol{j}\,c\left(1-\frac{v^2}{c^2}\right)^{1/2}\right\} \\ &= m\boldsymbol{v}\,c + \boldsymbol{j}\,m_0c^2 \\ &= \boldsymbol{M}\,c + \boldsymbol{j}\,m_0c^2\end{aligned} \tag{3.2}$$

Where

$\boldsymbol{v}$ is spatial vector velocity.
$\boldsymbol{M}$ is the spatial momentum vector of the particle energy mass.

Taking the magnitude of the final expression in (3.2) then gives

$$\left|\mathrm{E}\right|^2 = M^2c^2 + m_0^2c^4 \tag{3.3}$$

Where

M is the magnitude of the spatial momentum.

and (3.3) is the desired relationship.

With regard to the existence of energy as a spatial - temporal quantity, it is seen from the central expression in (3.2), that the temporal component is the rest mass energy while the spatial component is the spatial term in (3.2). However, this raises an apparent anomaly in that the spatial term in (3.2) does not appear to approximate to the classical expression for kinetic energy. The validity of (3.2), and therefore its use in the above derivation of Einstein's energy-momentum equation, can be verified in two ways.
First take the positive root of (3.3) thus

$$|\mathrm{E}| = \left(m^2v^2c^2 + m_0^2c^4\right)^{1/2} \tag{3.4}$$

Inserting [1], Eq.(3.6) for *m* gives

$$|\mathrm{E}| = \left\{\frac{m_0^2v^2c^2}{\left(1-\frac{v^2}{c^2}\right)} + m_0^2c^4\right\}^{1/2} \tag{3.5}$$

which reduces to

$$|\mathrm{E}| = \frac{m_0c^2}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} = mc^2 \tag{3.6}$$

The binomial expansion of which is

$$|\mathrm{E}| = m_0c^2 + \frac{m_0v^2}{2} - \frac{3}{8}\frac{m_0v^4}{c^2} + \ldots \tag{3.7}$$

Thus containing the rest mass energy and the classical expression for kinetic energy, (the remaining terms are relativistic additions to the kinetic energy).

A second verification of (3.2) can be effected as follows. In artificially induced motion it is well known that the rate of change of momentum is equal to the spatial distribution of energy. Thus taking spatial-temporal terms

$$\frac{d\mathrm{M}}{dt} = \frac{d\mathrm{E}}{ds} \tag{3.8}$$

Where

s is a distance along the Existence Velocity Vector path.

Thus inserting the momentum expression in (2.2) for $\mathbf{M}$ gives

$$\begin{aligned}\frac{d\mathrm{E}}{ds} &= \frac{d}{dt}\left[m\left\{\boldsymbol{v} + \boldsymbol{j}c\left(1-\frac{v^2}{c^2}\right)^{1/2}\right\}\right] \\ &= m\frac{d\boldsymbol{v}}{dt} + \boldsymbol{v}\frac{dm}{dt} + \boldsymbol{j}\left\{c\frac{dm}{dt}\left(1-\frac{v^2}{c^2}\right)^{1/2} - \frac{mv\,{dv}/{dt}}{c\left(1-\frac{v^2}{c^2}\right)^{1/2}}\right\}\end{aligned} \tag{3.9}$$

Converting this as follows

$$\frac{d\mathrm{E}}{ds}=m\frac{d\boldsymbol{v}}{ds}\frac{ds}{dt}+\boldsymbol{v}\frac{dm}{ds}\frac{ds}{dt}+\boldsymbol{j}\left\{c\frac{dm}{ds}\frac{ds}{dt}\left(1-\frac{v^2}{c^2}\right)^{1/2}-\frac{mv\frac{dv}{ds}\frac{ds}{dt}}{c\left(1-\frac{v^2}{c^2}\right)^{1/2}}\right\} \tag{3.10}$$

But by definition, (see [1])

$$\frac{ds}{dt}=c \tag{3.11}$$

and so (3.10) becomes

$$\frac{d\mathrm{E}}{ds}=mc\frac{d\boldsymbol{v}}{ds}+\boldsymbol{v}c\frac{dm}{ds}+\boldsymbol{j}\left\{c^2\frac{dm}{ds}\left(1-\frac{v^2}{c^2}\right)^{1/2}-\frac{mv\frac{dv}{ds}}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}\right\} \tag{3.12}$$

So that

$$\mathrm{E}\ =c\int\left(md\,\boldsymbol{v}+\boldsymbol{v}\,dm\right)+\boldsymbol{j}c^2\int\left\{\left(1-\frac{v^2}{c^2}\right)^{1/2}dm-\frac{mvdv}{c^2\left(1-\frac{v^2}{c^2}\right)^{1/2}}\right\} \tag{3.13}$$

Both the spatial and temporal terms in (3.13) are exact integrals, so that

$$\mathrm{E}\ =m\boldsymbol{v}c+\boldsymbol{j}\,m_0c^2 \tag{3.14}$$

as per (3.2). Its use in the above new simplified method of derivation of Einstein's relativistic mass - momentum relationship is therefore a valid one.

Another point which arises from the derivation in (3.2) is that the final result, (3.3), is the result of taking the magnitude of a spatial-temporal expression. It can therefore only have a positive root. Therefore, this suggests that Dirac's negative energy field does not exist in $\mathbf{D_0}$. This point is discussed further in Section 4.

3.2 The Spatial - Temporal Distribution of Energy.

Interpretation of the results of the previous Subsection is as follows. Consider a mass of matter in motion with a spatial velocity magnitude *v*.

Clearly, from (3.14) the total energy of the mass, is directed along its Existence Velocity Vector. This can be pictorially represented as in Fig. 3.1 below.

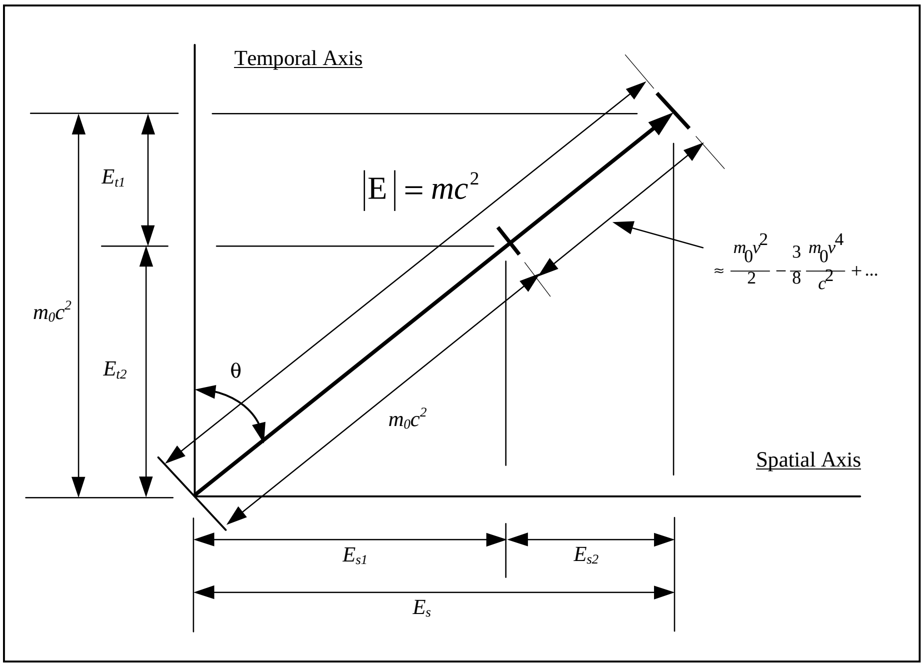

Fig. 3.1 Spatial Temporal Distribution of Matter Energy.

This representation is clear from (3.3), (3.6) and (3.7) where the magnitude of spatial - temporal terms about the Existence Velocity Vector have been taken to obtain the total energy. The spatial terms are therefore the projection of the total energy into the spatial dimension, i.e.

$$\begin{aligned} E_{s1} &= m_0c^2 \sin\theta = m_0vc \\ E_{s2} &= \left(\frac{m_0v^2}{2} - \frac{3}{8}\frac{m_0v^4}{c^2} + ...\right)\sin\theta = \frac{m_0v^3}{2c} + ... \end{aligned} \tag{3.15}$$

Where, from [1], ***sin*** θ = *v/c*, (note that this term is also present in (2.26)).

and therefore

$$\begin{aligned} E_s &= m_0vc + \frac{m_0v^3}{2c} + \dots \\ &= m_0vc\left(1 + \frac{v^2}{2c^2} + \dots\right) \\ &= m_0vc\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \\ &= mvc \end{aligned} \tag{3.16}$$

as in (3.2), (note that the vector designation in (3.2) has been dropped as unnecessary in this representation).

The temporal terms are in concert with the spatial terms a projection of the total energy into the temporal dimension i.e.

$$E_{t1}=\left(\frac{m_0v^2}{2}-\frac{3}{8}\frac{m_0v^4}{c^2}+...\right)\cos\theta=\left(\frac{m_0v^2}{2}-\frac{3}{8}\frac{m_0v^4}{c^2}+...\right)\left(1-\frac{v^2}{c^2}\right)^{1/2}$$
$$E_{t2}=m_0c^2\cos\theta=m_0c^2\left(1-\frac{v^2}{c^2}\right)^{1/2} \qquad (3.17)$$

Where, from [1], $\boldsymbol{cos\,\theta}=\left(1-\frac{v^2}{c^2}\right)^{1/2}$, (note that this term is also present in (2.26)).

and therefore

$$\begin{aligned}E_t&=\left(m_0c^2+\frac{m_0v^2}{2}-\frac{3}{8}\frac{m_0v^4}{c^2}+...\right)\left(1-\frac{v^2}{c^2}\right)^{1/2}\\&=m_0c^2\end{aligned} \qquad (3.18)$$

Thus it is shown that although the accelerative force, and therefore the applied energy in such motion is all derived spatially, the distribution of this energy is shared between the spatial and temporal dimensions. The same is true of the rest mass energy as the spatial velocity increases and the Existence Velocity Vector rotates into the spatial dimension. The projection of the total energy into the temporal dimension contains an element of the kinetic energy, which compensates for the reduction in rest mass energy in that direction, so that the total energy in that dimension remains constant. This is necessarily so because as stated above all of the accelerating energy is directed in the spatial dimension, and no energy is either added or subtracted in the temporal direction. The energy projected into the spatial dimension increases as the spatial velocity increases but, as shown also consists of both kinetic and rest mass energy elements as in the temporal dimension.

It must be noted that despite this spatial-temporal distribution, the energy actually "perceived" in the spatial direction is that of the total energy and not just that reflected into it. That this is so evident from the fact that the total energy of the particle can be derived from just spatial considerations as well as from a combination of both spatial and temporal. However, it is believed that despite this "perception", the energy actually "available" in the spatial dimension is only that reflected into it. This is still greater than the energy supplied to cause the motion, and therefore may have interesting consequences at the quantum level, i.e. a different explanation for quantum tunnelling.

4.0 Conclusions.

There are several significant results that have emerged from the analytical processes pursued in this paper. The most important is that, accepting both Planck's and de Broglie's hypotheses, it has been shown that a matter particle possessing a spatial velocity can exhibit a dual existence not only in the spatial, but also in the temporal dimension of $\mathbf{D_0}$, in both corpuscular and wave function form.

The spatial component of the dual existence, as determined herein, is particularly significant for two reasons. Firstly, it obviously supports this effect as the explanation for the fact that electrons, protons, neutrons and even molecular particles can produce interference patterns when fired through an appropriately sized lattice grid, in exactly the same manner as electromagnetic radiation. Secondly, and perhaps more importantly, the spatial matter wave component, when considered as that of an electron in a Bohr atom, will provide theoretical justification of the quantisation process of the orbit energy levels in that theory. This will permit the resurrection of the Bohr/Sommerfeld theory of atomic structure, as an alternative to that of modern quantum mechanics. This will form the subject of a future series of papers.

The remaining conclusions are discussed in order of appearance in the text.

The derivation of the matter wave characteristics of a stationary particle showed that this wave propagates entirely along the temporal axis. This was also effectively demonstrated in the discussion on energy where in (3.2) if the spatial velocity is put to zero, the total energy of the stationary particle, the rest mass energy, is seen to be all temporal. This explains why the vast amount of energy contained within matter cannot presently be tapped in the spatial dimension. It also explains why the possible future extraction of this energy, as demonstrated in [3], would require the application of a temporal force. However, when a mass is in spatial motion and its Existence Velocity Vector has rotated towards the spatial dimension, it is seen that the energy projected into the spatial dimension is greater than just the kinetic energy acquired due to the motion. Perhaps this may, eventually, provide easier access to the rest mass energy of matter.

The analysis and discussion on the spatial - temporal distribution of energy in this paper has been somewhat superficial because the main subject here was the characteristics of de Broglie matter waves in Pseudo-Euclidean Space-Time. The two subjects are related via Planck's quantum energy hypothesis. A more in depth dissertation on the spatial - temporal distribution of energy will be the subject of a future paper.

The Compton effect confirms that light certainly conforms to Planck's hypothesis of energy quanta, but also confirms that it can exhibit the characteristics of a particle, i.e. a photon. The analysis here agrees with other approaches, in appearing to necessitate that the photon possesses zero rest mass energy, in order for its propagation velocity to avoid a relativistic increase in mass. However, it is considered that this creates an anomaly, because, if the photon is to be regarded as a particle as well as an electromagnetic radiation wave function, then it must possess a mass in accordance with Einstein's classic energy-mass relationship. This anomaly is however, removed if the mechanism causing photon emission is via a natural application of a combined spatial/temporal accelerative force, constant in magnitude, but with a spatial/temporal direction vector that was a function of its spatial velocity. In this case its mass rate variability would be zero. Such a process was the detailed subject of [3] Section 2.3. As a consequence, the photon could possess a rest mass energy given by the second term on the RHS of the first expression in (2.10). This energy would be completely converted to kinetic energy via the process of [3] Section 2.3 when photons attain light velocity. Whether they can exist as free particles in a spatially stationary state, or at velocities lower than that of light is unknown, but if so, they may well exhibit the characteristics of particles similar to the neutrino.

The manner in which kinetic energy was stored by an accelerated mass was, as in the literature and in [1], shown to be the relativistic increase in mass. The actual method of storage was not addressed. This paper has however, shown that this method is by an increase in the matter wave frequency of the body in motion. The frequency in question is that of the matter wave propagating along the path of the body's Existence Velocity Vector, and the projection of this into the spatial dimension. The storage of kinetic energy by this means is, on the basis of Planck's and de Broglie's hypotheses, believed to be the final answer to this issue.

The clarification provided for the so called phase velocities of matter waves, has it is believed, removed an awkward anomaly. Einstein's criterion of the maximum spatial velocity of propagation would be contravened, as would the primary criterion of existence in $\mathbf{D_0}$, if these parameters truly existed. This is irrespective of statements such that these terms have no physical meaning. These parameters do not exist for the reason stated herein, and have been replaced by the projected terms as shown and discussed.

When taking the root of a squared term, of course both positive and negative results are mathematically permissible. But this is a mathematical concept only and may not apply to the physics of the problem depicted. The derivation of Einstein's relativistic energy - momentum relationship presented herein, has shown that it contains both spatial and temporal components. The mathematical magnitude of this relationship therefore can only have a positive value, because both the spatial and temporal components are positive. The consequence of this is that within the Relativistic Space-Time Domain that is $\mathbf{D_0}$, Dirac's 'sea of negative energy' cannot exist. In [1] $\mathbf{D_0}$ was shown to be in fact, Pseudo-Euclidean Space-Time and therefore, gravitation apart, the Domain in which all things, down to atomic level and below, exist. Consequently, if accepted, this would have serious implications in the field of quantum electrodynamics.

Finally, there is the question of the nature of matter waves themselves. It has been stated, [4], that matter waves are distinctly different from electromagnetic waves and propagate at different, (lower), velocities. However, this reference unfortunately contains a number of grammatical and mathematical errors and may not be a reliable reference. The presentation here indicates that matter waves propagate at the velocity of light. Their precise nature however, remains unclear.

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