RESURRECTION OF THE BOHR/SOMMERFELD THEORY OF ATOMIC STRUCTURE.

[6]

THE ADDITION OF SMALL RELATIVISTIC CORRECTION TERMS TO ELECTRON MAGNETIC DIPOLE COUPLING ENERGIES, et al.

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ABSTRACT.

The resurrected Bohr/Sommerfeld theory of atomic structure, as so far re-stated in [1], [2], [3], [4], and [5], is herein further developed to incorporate small relativistic additions to the electron's magnetic dipole coupling energies, both orbital and spin.
These additions involve determination of the effects of the distributed nature of electric charge, the distributed nature of mass, and the relativistic mass increase due to orbital and spin velocities.
The fine resolution of air refraction is also addressed as is the optimisation of the electron spin matter wave radius.

CONTENTS.

1.0 Introduction.

In the Discussion of Results in [4], it was stated that the hydrogen emission spectra and orbital energy levels and intervals, could possibly be further improved in comparison with those in [6], by considering three further aspects. They are:-

(i) Incorporation of the effects of, electron distributed surface charge, distributed mass and relativistically increased mass, into the orbital and spin magnetic dipole coupling energies.

(ii) Improved resolution of the air refractive index, Π_{air}, and its incorporation in the determination of the air spectra in orbital shells 2 to 8.

(iii) Subsequent to (i) and (ii), optimisation of the spin matter wave radius, Γ_e, in all orbital shells, 2 to 8, to meet the spin angular momentum criteria in the non-transitional orbitals, and further improvement of emission spectral results in the transitional shells.

All three of the above potential improvements are extensively explored in this paper. In addition, subsequent to (iii) above, the relationship of the electron spin matter wave radius to its principle quantum number, *n*, its azimuthal quantum number n_ϕ^*, and its spin quantum number ${}_e n_{sp}$, is developed and discussed.

In the interests of continuity, the detailed mathematical derivations involved in the developments of (i) to (iii) above, have been relegated to the Appendices with only the results summarised in the main text for discussion.

Finally, it is clear that this paper follows on from those in the References, and therefore in the mathematical derivations, a parameter will only be defined if it has not already been so in these earlier papers.

2.0 Relativistic Additions to Electron Magnetic Coupling Energies.

2.1 Preamble.

To determine these relativistic terms, it is first necessary to derive a number of preliminary parameters. The first is a term for the semi-latus rectum of the basic electron orbit, taking into account the distributed nature of electron mass. This can then be incorporated into a specific expression for the relativistic modifier term, $\left(1 - v^2/c^2\right)^{1/2}$ that is required in the enhanced electron orbital magnetic moment.

In parallel with the above process, a separate relationship is developed for the relativistically adjusted orbital angular momentum, into which have been incorporated the effects of distributed electron mass.

The results of the above two subsidiary developments, are then incorporated into a new expression for the electron orbital magnetic moment, which in turn is then used, in conjunction with that of the proton, to produce an updated expression for the net orbital magnetic moment.

To accomplish the changes for the electron spin magnetic dipole, two analyses are required, one for the non-transitional orbitals in which the electron spin angular momentum quantum criteria is met, and one for transitional orbitals in which the conditions for an orbit transition occur, before the spin angular momentum quantum criteria is met. These analyses produce two new expressions for the electron spin magnetic dipole applicable to the two orbital scenarios above.

The results of all the above derivations are then coupled appropriately to produce modified coupling energies, which are finally added into the results from previous papers, to produce a new final orbital energy relationship for both non-transitional and transitional states.

2.2 The Orbital Magnetic Dipole Moment.

From Appendix A, Eq.(A.27), the electron orbital magnetic dipole moment with the relativistic additions incorporated is given by

$$ {}_e\Psi^*_{OR} = \frac{Zehn_j}{4\pi cm_e}\left\{1-\frac{\kappa^2 Z^2}{n^2}\left(\frac{2n^2}{n^{*2}_\phi}-1\right)\right\} \tag{2.1}$$

This may be compared with [3], Eq.(B.7) which shows that the modification term is that inside the main bracket. Therefore, from [3], Eq.(2.3) and (2.1) above, the net orbit magnetic dipole is now given by

$$ \Psi^*_{OR} = {}_e\Psi^*_{OR} + {}_p\Psi_{OR} = \frac{Zehn_j}{4\pi cm_e}\left\{1-\frac{\kappa^2 Z^2}{n^2}\left(\frac{2n^2}{n^{*2}_\phi}-1\right)-\frac{\gamma_p m_e}{m_p}\right\} \tag{2.2}$$

This may be compared with [3], Eq.(3.27) to see the presence of the additional terms.

2.3 The Spin Magnetic Dipole Moment.

2.3.1. Non-Transitional Orbitals.

These orbitals are 1s(+) and 2s(+) and from Appendix B.1, Eq.(B.7), the electron spin magnetic dipole moment for these orbitals is given by

$$ {}_e\Psi^*_{sp} = \frac{{}_e n_{sp} he}{2\pi cm_e\left(1+\dfrac{9\,{}_e n^2_{sp} h^2}{20\pi^2 c^2 \Gamma^2_e m^2_e}\right)^{1/2}} \tag{2.3}$$

This may be compared with [3], Eq.(2.1) which shows that the modification term is the square root term in the denominator.

2.3.2. Transitional Orbitals.

These orbitals are those other than 1s(+) and 2s(+) and from Appendix B, Eq.(B.8) the electron spin magnetic dipole moment for these orbitals is given simply by

$$ {}_e\Psi^*_{sp} = \frac{e\Gamma_e}{3} \tag{2.4}$$

2.4 Total Orbital Energy.

2.4.1. The Coupling Energy of the Electron Spin Dipole and the Net Orbit Dipole.

2.4.1.1. Non-Transitional Orbitals.

From Appendix C, Eq.(C.5) the orbital energy generated as a result of the coupling between the net orbit magnetic dipole and that resulting from electron spin for these orbitals is given by

$$ {}_{e}E_{SO}(OR,NT)=\frac{\dfrac{hR_{hy}Z^2}{n^2}\dfrac{\kappa^2Z^2}{n^2}\left\{1-\dfrac{\kappa^2Z^2}{n^2}\left(\dfrac{2n^2}{n_\phi^{*2}}-1\right)-\dfrac{\gamma_p m_e}{m_p}\right\}}{\left(1+\dfrac{9\,{}_{e}n_{sp}^2h^2}{20\pi^2c^2\Gamma_e^2m_e^2}\right)^{1/2}}\frac{m_e}{m_0}\frac{n\,{}_{e}n_{sp}}{n_j^2} \qquad (2.5) $$

This may be compared with [3], Eq.(3.34).

2.4.1.2. Transitional Orbitals.

From Appendix C, Eq.(C.3), the orbital energy generated as a result of the coupling between the net orbit magnetic dipole and that resulting from electron spin for these orbitals is given by

$$ {}_{e}E_{SO}(OR,T)=\frac{hR_{hy}Z^2}{n^2}\frac{\kappa^2Z^2}{n^2}\left\{1-\frac{\kappa^2Z^2}{n^2}\left(\frac{2n^2}{n_\phi^{*2}}-1\right)-\frac{\gamma_p m_e}{m_p}\right\}.\frac{2\pi m_e^2cn\Gamma_e}{3hm_0n_j^2} \qquad (2.6) $$

2.4.2. **The Coupling Energy of the Electron Spin Dipole and the Proton Spin Dipole.**

2.4.2.1. Non-Transitional Orbitals.

From Appendix C, Eq.(C.9), the orbital energy generated as a result of the coupling between the magnetic dipoles due to electron and proton spin for these orbitals is given by

$$ {}_{e,p}E_{ss}(OR,NT)=-\frac{hR_{hy}Z^2}{n^2}\frac{\kappa^2Z^2}{n^2}\frac{2m_e^2\gamma_p\delta_p}{m_0m_p}\frac{n\,{}_{e}n_{sp}\,{}_{p}n_{sp}}{n_\phi^*n_j^2\left(1+\dfrac{9\,{}_{e}n_{sp}^2h^2}{20\pi^2c^2\Gamma_e^2m_e^2}\right)^{1/2}} \qquad (2.7) $$

This may be compared with [3], Eq.(3.38).

2.4.2.2. Transitional Orbitals.

From Appendix C, Eq.(C.7), the orbital energy generated as a result of the coupling between the magnetic dipoles due to electron and proton spin for these orbitals is given by

$$ {}_{e,p}E_{ss}(OR,T)=-\frac{hR_{hy}Z^2}{n^2}\frac{\kappa^2Z^2}{n^2}\frac{2m_e^2\gamma_p\delta_p}{m_0m_p}.\frac{2\pi m_ec\Gamma_e}{eh\,{}_{e}n_{sp}}\frac{n\,{}_{e}n_{sp}\,{}_{p}n_{sp}}{n_\phi^*n_j^2} \qquad (2.8) $$

2.4.3. **Total Orbital Energy.**

2.4.3.1. Non-Transitional Orbitals.

The total energy in the orbit for these orbitals is now obtained by modifying [4], Eq.(4.2) with (2.5) and (2.7) above to give

$$
{}_eE_{OR}(NT)=-\frac{hR_{hy}Z^2}{n^2}\left[1+\frac{\kappa^2Z^2}{n^2}\left\{\begin{array}{l}\dfrac{n}{n_\phi^*}-\dfrac{m_e}{m_0}\left\{\begin{array}{l}\left[1-\dfrac{\kappa^2Z^2}{n^2}\left(\dfrac{2n^2}{n_\phi^{*2}}-1\right)-\dfrac{\gamma_p m_e}{m_p}\right]\dfrac{{}_en_{sp}}{n_\phi^* n_j}+\\ \dfrac{2m_e^2}{m_0m_p}\gamma_p\delta_p\dfrac{{}_en_{sp}\,{}_pn_{sp}}{n_\phi^* n_j^2}\end{array}\right\}\left(1+\dfrac{9\,{}_en_{sp}^2h^2}{20\pi^2c^2\Gamma_e^2m_e^2}\right)^{-1/2}\\ -\dfrac{m_e^2}{2\kappa^2m_p^2Z^2}\dfrac{n^3}{n_j}-\dfrac{5\pi^2m_e^2c^2\Gamma_e^2}{3h^2}\dfrac{n}{n_j^3}-\dfrac{3}{4}\end{array}\right\}\right] \quad (2.9)
$$

2.4.3.2. Transitional Orbitals.

The total energy in the orbit for these orbitals is now obtained by modifying [4], Eq.(4.2) with (2.6) and (2.8) above to give

$$
{}_eE_{OR}(T)=-\frac{hR_{hy}Z^2}{n^2}\left[1+\frac{\kappa^2Z^2}{n^2}\left\{\begin{array}{l}\dfrac{n}{n_\phi^*}-\dfrac{m_e}{m_0}\left\{\begin{array}{l}\left[1-\dfrac{\kappa^2Z^2}{n^2}\left(\dfrac{2n^2}{n_\phi^{*2}}-1\right)-\dfrac{\gamma_p m_e}{m_p}\right]\dfrac{n}{n_j^2}+\\ \dfrac{2m_e^2}{m_0m_p}\gamma_p\delta_p\dfrac{n\,{}_pn_{sp}}{n_\phi^* n_j^2}\end{array}\right\}\dfrac{2\pi m_ec\Gamma_e}{h}\\ -\dfrac{m_e^2}{2\kappa^2m_p^2Z^2}\dfrac{n^3}{n_j}-\dfrac{5\pi^2m_e^2c^2\Gamma_e^2}{3h^2}\dfrac{n}{n_j^3}-\dfrac{3}{4}\end{array}\right\}\right] \quad (2.10)
$$

Eqs.(2.9) and (2.10) have now been built into the EXCEL spreadsheet HydrogenSpectra.XLS to calculate detailed orbital energy levels, energy intervals and hydrogen spectra etc.

2.5 Improvement of the Resolution of the Air Refractive Index for the Hydrogen Air Spectra.

Examination of the amended spectral results showed that the maximum interval occurred in shell 8 and was 2.1Å at approximately 19,0556 Å. The variation in air refractive index over this interval was too small to affect the results determined here.

2.6 Optimisation of the Spin Matter Wave Radius.

Subsequent to the implementation of the amendments in Sections 2.4 and 2.5, the value of spin matter wave radius was optimised to obtain the best possible results in HydrogenSpectra.XLS for the following parameters

(i) Vacuum Spectra.
(ii) Orbital Energy Levels.
(iii) Orbital Energy Intervals, (Lamb Shift).
(iv) Air Spectra.

The results showed considerable improvement over those presented in [5] and are discussed in detail below.

3.0 Discussion of Results.

3.1 Vacuum Spectra.

In [5] there was full agreement with [6] for the vacuum emission spectra wavelengths to four decimal places, (this was without full optimisation of the spin matter wave radius). The maximum error was 0.00006Å, with all applicable orbitals exhibiting a ± error of a similar magnitude.

In the modified version here, the only orbitals showing a vacuum emission spectra wavelength difference to [6] are 6p(+), (0.00002Å) and 6p(-), (0.00004Å), all others being zero to five decimal places.

3.2 Energy Levels.

In [5] the maximum energy level difference compared to [6] was -0.0503cm^{-1} {8k(-)}, but there was also a negative systemic difference that averaged -0.0445cm^{-1} over all orbitals in shells 2 to 8.

In the amended version here, the maximum difference is -0.0060cm^{-1}, {4p(+)}. The systemic difference has been eliminated and the average difference over all orbitals is -0.0007cm^{-1}.

3.3 Energy Intervals.

The Lamb Shift intervals in [5] was shown in [5] Table 5.1 where there was full agreement with [6] in shells 2 and 8, but a small difference was apparent in shells 3 to 7.

In the modified version here all lamb Shift intervals in all shells 2 to 8 now agree fully with those in [6].

3.4 Air Spectra.

In [5] the maximum difference to [6] in the air spectral wavelengths was 0.2644Å, {8k(+) to 7i(+)}. Also in 90% of all other orbital air transitions the difference was positive.

In the modified version here the maximum difference is -0.1572Å, {6s(+) to 5p(+)}. The difference distribution throughout all other applicable transitions has been improved to 75% positive.

3.5 Other Results.

In this paper the ground level energy has become 109,675.8909cm^{-1} as against 109,678.7135cm^{-1} in [5]. Also ionisation from ground level has become 13.59808237eV as against 13.59843233eV in [5]. The Lamb Shift frequency of 1057.77Mc/s is unchanged between the two papers.

3.6 The Spin Matter Wave Radius.

Clearly the most contentious part of this resurrected theory, is the proposal that the electron's spin matter wave radius be a variable parameter. As was discussed at some length in [5], this concept was introduced to provide a mechanism by which the electron spin angular momentum criteria could be met in orbitals 1s(+) and 2s(+). In all other orbitals a transition occurs before this criteria, is satisfied, but the mechanism was still active.

Now that all possible electron effects have been included in this resurrected theory, (at least to a second order relativistic level), it is accordingly necessary to show that this parameter is more than just an adjustment factor, and that it obeys appropriate relationships to the main quantum numbers extant in the theory.

In Appendix D, the values of Γ_e in shells 1 to 8, have been tabulated and graphed against the primary quantum number, n, for applicable values of the azimuthal quantum number n_ϕ^*, and the electron spin quantum number ${}_e n_{sp}$. These plots exhibit a very precise form and exact curve fitting produces the following results. The curves obey the following algorithm

$$\Gamma_e = an^3 + bn^2 + cn + d \qquad (3.1)$$

Where

Γ_e is the electron spin matter wave radius.
n is the primary quantum number.

The value of the coefficients in (3.1) are given in Appendix D Table D.2. This table and the plots (Figs.D.1 and D.2), show that Γ_e bears a very precise relationship to the primary, azimuthal and electron spin quantum numbers. From the data presented, it should be possible to predict the values of the coefficients in (3.1), in order to determine the value of Γ_e in any orbit shell for any applicable orbital.

4.0 Conclusions.

With the incorporation of the final effects due to the electron, the results now show exemplary agreement with the data in [6]. The only area where there is some minor disagreement is in the air spectra. However, as was stated in a previous paper, the air refractive index used to refine these spectra was determined via calculation from [7]. A more reliable method of determining this parameter to the necessary precision, may yet further improve these results.

The demonstration that the spin matter wave radius of the electron follows a very precise relationship to the primary, azimuthal and electron spin quantum numbers, shows that this parameter is not just an "adjustment factor". It is clearly one of the principle mechanisms in the cause of electron transitions, and in determining the demarcation between transitional and non-transitional orbitals, as well as instrumental in ensuring the spin angular momentum criteria is met in the non-transitional orbitals.

Further development of this resurrected theory, could be pursued by treating the proton with the same degree of detail as has been afforded the electron in this paper. The improvement in results would however be minimal, and would require expression to a higher degree of precision, (~ 6 decimal places). Whilst from a purists point of view, such an exercise may be desirable, i.e. obtaining a more complete version of the orbital energy equations, the fine detail of the improvement in results, would only be truly meaningful if there were empirical measurements of the appropriate accuracy and precision, against which they could be compared.

APPENDIX A.

Detailed Mathematical Derivation of the Electron Orbital Magnetic Dipole.

To effect this derivation, it is first necessary to determine two subsidiary terms. The first is an expression for the orbital angular momentum of the basic orbit but including the effect of distributed mass. The second is an approximate expression for the relativistic modifier $\left(1-{v^2}/{c^2}\right)^{1/2}$.

A.1 Derivation of Electron Orbital Angular Momentum with Distributed Mass.

Consider Fig. A.1,

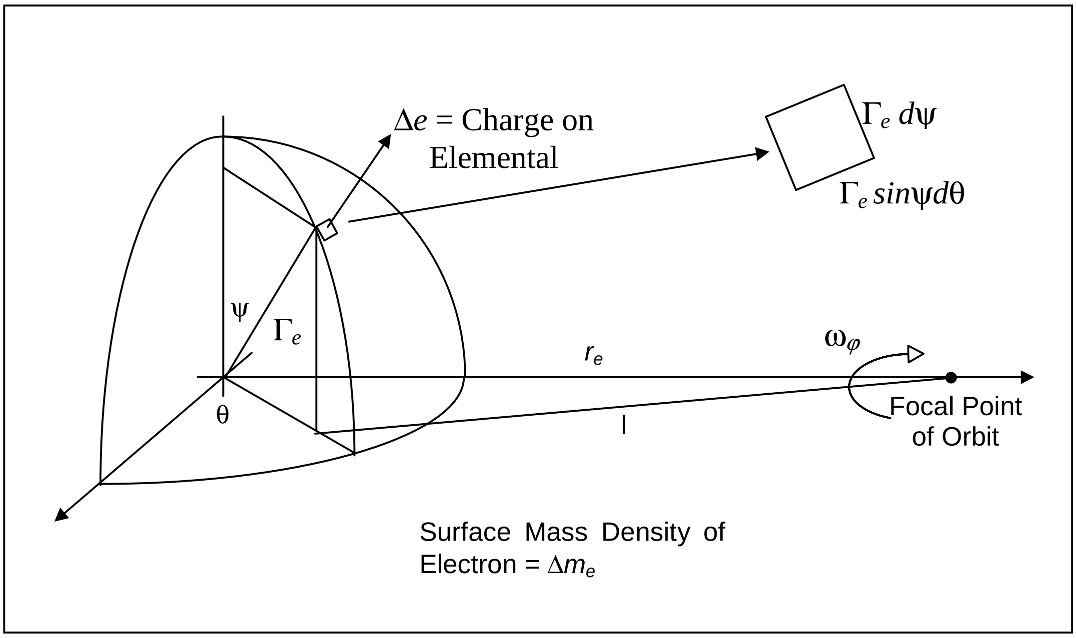

Fig. A.1 - Electron with Distributed Mass.

First, developing the angular momentum in the basic orbit, i.e. without relativistic mass increase.

$$\Delta M_{\varphi} = \Delta m_e \omega_{\varphi} l^2 \Gamma_e^2 \, \boldsymbol{sin}\,\psi d\psi d\theta \qquad \text{(A.1)}$$

The length l, the distance of the elemental from the from the focal point of the orbit is

$$l^2 = r_e^2\left(1 - 2\frac{\Gamma_e}{r_e}\boldsymbol{sin}\,\psi\,\boldsymbol{sin}\,\theta + \frac{\Gamma_e^2}{r_e^2}\boldsymbol{sin}^2\,\psi\right) \qquad \text{(A.2)}$$

Inserting (A.2) into (A.1) and integrating over the surface of the electron

$$M_{\varphi} = \Delta m_e \omega_{\varphi} r_e^2 \Gamma_e^2 \int_{\theta=0}^{2\pi}\int_{\psi=0}^{\pi}\left(\boldsymbol{sin}\,\psi - 2\frac{\Gamma_e}{r_e}\boldsymbol{sin}^2\,\psi\,\boldsymbol{sin}\,\theta + \frac{\Gamma_e^2}{r_e^2}\boldsymbol{sin}^3\,\psi\right)d\psi d\theta \qquad \text{(A.3)}$$

and this evaluates to

$$M_\varphi = m_e \omega_\varphi r_e^2 \left(1 + \frac{2\Gamma_e^2}{3r_e^2}\right) \tag{A.4}$$

where the distributed mass effect is represented by the term in the brackets. Incorporating the relativistic mass increase effect then gives

$$M_\varphi^* = \frac{m_e \omega_\varphi r_e^2 \left(1 + \frac{2\Gamma_e^2}{3r_e^2}\right)}{\left(1 - v^2/c^2\right)^{1/2}} \tag{A.5}$$

This is a simplistic manner of introducing relativistic mass increase to this expression, but is acceptable for the level of accuracy and precision targeted.

Eq.(A.5) will be used in the derivation of the magnetic dipole below. Prior to that, an expression for the relativistic modifier is required.

A.2 Derivation of a Suitable Expression for the Relativistic Modifier.

An expression for this modifier is required in terms of the orbital parameters of the electron. To effect this it is first necessary to obtain an expression for the semi-latus rectum of the basic orbit taking account of distributed mass.

From [1], Eq.(A.8), (*h* here is the swept area of the orbit)

$$h = \frac{\omega_\varphi r_e^2}{\left(1 - v^2/c^2\right)^{1/2} \left(1 - \frac{F_0^2}{m_e^2 c^2 h^2}\right)^{1/2}} \tag{A.6}$$

Substituting from (A.5) for $\omega_\varphi r_e^2$ gives

$$h = \frac{M_\varphi^*}{m_e \left(1 + \frac{2\Gamma_e^2}{3r_e^2}\right) \left(1 - \frac{F_0^2}{m_e^2 c^2 h^2}\right)^{1/2}} \tag{A.7}$$

Solving for *h* gives

$$h = \frac{M_\varphi^*}{m_e} \left[\frac{1}{\left(1 + \frac{2\Gamma_e^2}{3r_e^2}\right)} + \frac{F_0^2}{c^2 M_\varphi^{*2}} \right]^{1/2} \tag{A.8}$$

and the effect of the distributed mass can be seen by comparing (A.8) with [1], Eq.(A.10).

Substituting (A.8) into [1], Eq.(A.6) then gives for *L*

$$L=\frac{1}{m_0F_0}\left[\frac{M_\varphi^{*2}}{\left(1+\frac{2\Gamma_e^2}{3r_e^2}\right)^2}+\frac{F_0^2}{2c^2}\left(1-\varepsilon^2\right)\right] \tag{A.9}$$

and inserting Ze^2 for F_0 gives the desired expression for *L* as

$$L=\frac{M_\varphi^{*2}}{m_0Ze^2}\left[\frac{1}{\left(1+\frac{2\Gamma_e^2}{3r_e^2}\right)^2}+\frac{Z^2e^4}{2c^2M_\varphi^{*2}}\left(1-\varepsilon^2\right)\right] \tag{A.10}$$

The derivation of the relativistic modifier can now be developed. Starting from [1], Eq.(2.19)

$$v^2=\dot{r}_e^2+\omega_\phi^2r_e^2=\frac{\omega_\varphi^2L^2\varepsilon^2\,\boldsymbol{sin}^2\varphi}{\left(1+\varepsilon\,\boldsymbol{cos}\,\varphi\right)^4}+\frac{\omega_\varphi^2L^2}{\left(1+\varepsilon\,\boldsymbol{cos}\,\varphi\right)^2}\left(1+\frac{Z^2e^4}{c^2M_\varphi^{*2}}\right) \tag{A.11}$$

Solving this for the required relativistic modifier, gives after some reduction in which only second order relativistic terms are retained.

$$\left(1-\frac{v^2}{c^2}\right)^{1/2}=1-\frac{M_\varphi^{*2}\left(1+2\varepsilon\,\boldsymbol{cos}\,\varphi+\varepsilon^2\right)}{m_e^2L^2c^2\left(1+\frac{2\Gamma_e^2}{3r_e^2}\right)^2} \tag{A.12}$$

where $\omega_\varphi^2r_e^4$ has been substituted from a re-arranged (A.5). Taking the average around the orbit, (A.12) becomes

$$\left(1-\frac{v^2}{c^2}\right)^{1/2}=1-\frac{M_\varphi^{*2}\left(1+\varepsilon^2\right)}{m_e^2L^2c^2\left(1+\frac{2\Gamma_e^2}{3r_e^2}\right)^2} \tag{A.13}$$

Substitution from (A.10) for *L* then gives after considerable reduction, again in which only second order relativistic terms are retained, (which results in the disappearance of the term representing the distributed mass effect),

$$\left(1-\frac{v^2}{c^2}\right)^{1/2}=1-\frac{Z^2e^4\left(1+\varepsilon^2\right)}{c^2\left\{M_\varphi^{*2}+\frac{Z^2e^4}{c^2}\left(1-\varepsilon^2\right)\right\}} \tag{A.14}$$

From [1], Eq.(3.44)

$$M_\varphi^* = \frac{nh}{2\pi}\left(1 - \frac{\kappa^2 Z^2}{2nn_\phi^*}\right)\left(1-\varepsilon^2\right)^{1/2} \qquad \text{(A.15)}$$

Insertion of this into (A.14) gives

$$\left(1-\frac{v^2}{c^2}\right)^{1/2} = \frac{\frac{\kappa^2 Z^2}{n^2}\frac{\left(1+\varepsilon^2\right)}{\left(1-\varepsilon^2\right)}}{1-\frac{\kappa^2 Z^2}{n^2}\left(\frac{n}{n_\varphi^*}-1\right)} \qquad \text{(A.16)}$$

and with

$$1-\varepsilon^2 = \frac{n_\varphi^{*2}}{n^2} \qquad \textit{so that} \qquad 1+\varepsilon^2 = 2-\frac{n_\varphi^{*2}}{n^2}$$

Eq.(A.16) finally reduces to

$$\left(1-\frac{v^2}{c^2}\right)^{1/2} = 1-\frac{\kappa^2 Z^2}{n^2}\left(\frac{2n^2}{n_\varphi^{*2}}-1\right) \qquad \text{(A.17)}$$

A.3 Derivation of the Orbital Magnetic Dipole.

Consider again Fig. A.1. The charge on the elemental is

$$\Delta e = \frac{Ze\,\boldsymbol{sin}\,\psi d\psi d\theta}{4\pi} \qquad \text{(A.18)}$$

If i = Current Flow = velocity x charge/path, then

$$i = \frac{Z\omega_\varphi e\,\boldsymbol{sin}\,\psi d\psi d\theta}{8\pi^2} \qquad \text{(A.19)}$$

The elemental magnetic moment is then

$$\Delta_e\Psi_{OR} = \frac{Ai}{c} = \frac{Ze\omega_\varphi l^2\,\boldsymbol{sin}\,\psi d\psi d\theta}{8\pi c} \qquad \text{(A.20)}$$

Inserting (A.2) for l^2 and integrating over the surface of the electron then gives

$${}_e\Psi_{OR} = \frac{Ze\omega_\varphi}{8\pi c}\int_{\theta=0}^{2\pi}\int_{\psi=0}^{\pi}\left(r_e^2\,\boldsymbol{sin}\,\psi - 2r_e\Gamma_e\,\boldsymbol{sin}^2\,\psi\,\boldsymbol{sin}\,\theta + \Gamma_e^2\,\boldsymbol{sin}^3\,\psi\right)d\psi d\theta \qquad \text{(A.21)}$$

and this evaluates to

$$ {}_e\Psi_{OR} = \frac{Ze\omega_\varphi r_e^2}{2c}\left(1+\frac{2\Gamma_e^2}{3r_e^2}\right) \tag{A.22} $$

Now substituting from (A.5) for $\omega_\varphi r_e^2$ gives

$$ {}_e\Psi_{OR}^* = \frac{ZeM_\varphi^*\left(1-{v^2}/{c^2}\right)^{1/2}}{2cm_e} \tag{A.23} $$

and note that the effects of distributed surface charge and distributed mass have cancelled. Now

$$ M_\varphi^* = \frac{n_\varphi^* h}{2\pi} \tag{A.24} $$

so that

$$ {}_e\Psi_{OR}^* = \frac{Zehn_\varphi^*}{4\pi cm_e}\left(1-\frac{v^2}{c^2}\right)^{1/2} \tag{A.25} $$

and now note that this is identical to [3], Eq.(B.6) but with the addition of the relativistic modifier. Insertion of (A.17) for that term then gives

$$ {}_e\Psi_{OR}^* = \frac{Zehn_\varphi^*}{4\pi cm_e}\left\{1-\frac{\kappa^2 Z^2}{n^2}\left(\frac{2n^2}{n_\varphi^{*2}}-1\right)\right\} \tag{A.26} $$

which in a fully coupled environment finally becomes

$$ {}_e\Psi_{OR}^* = \frac{Zehn_j}{4\pi cm_e}\left\{1-\frac{\kappa^2 Z^2}{n^2}\left(\frac{2n^2}{n_\varphi^{*2}}-1\right)\right\} \tag{A.27} $$

APPENDIX B.

Detailed Mathematical Derivation of Electron Spin Magnetic Dipole.

The spin angular momentum of the electron incorporating relativistic effects is given by [2], Eq(A.9). Viz.

$$M_{sp}^{*} = \frac{\frac{2}{3} m_e \, {}_e\omega_{sp}\Gamma_e^2}{\left(1 - \frac{4\,{}_e\omega_{sp}^2 r_e^2}{5c^2}\right)^{1/2}} \tag{B.1}$$

and from [3], Eq. (B.12),

$$ {}_e\Psi_{sp} = \frac{{}_e\omega_{sp}\Gamma_e^2 e}{3c} \tag{B.2}$$

Now solving (B.1) for ${}_e\omega_{sp}\Gamma_e^2$ gives

$$ {}_e\omega_{sp}\Gamma_e^2 = \frac{3M_{sp}^{*}}{2m_e\left(1 + \frac{9M_{sp}^{*2}}{5c^2\Gamma_e^2}\right)^{1/2}} \tag{B.3}$$

Now

$$M_{sp}^{*} = \frac{{}_e n_{sp} h}{2\pi} \tag{B.4}$$

which when substituted into (B.3) gives

$$ {}_e\omega_{sp}\Gamma_e^2 = \frac{3\,{}_e n_{sp} h}{4\pi m_e\left(1 + \frac{9\,{}_e n_{sp}^2 h^2}{20c^2\Gamma_e^2 m_e^2}\right)^{1/2}} \tag{B.5}$$

and substituting this into (B.2) then gives

$$ {}_e\Psi_{sp}^{*} = \frac{{}_e n_{sp} he}{4\pi c m_e\left(1 + \frac{9\,{}_e n_{sp}^2 h^2}{20\pi^2 c^2\Gamma_e^2 m_e^2}\right)^{1/2}} \tag{B.6}$$

which in a coupled environment becomes

$$ {}_e\Psi_{sp}^* = \frac{{}_e n_{sp} he}{2\pi c m_e \left(1 + \frac{9\,{}_e n_{sp}^2 h^2}{20\pi^2 c^2 \Gamma_e^2 m_e^2}\right)^{1/2}} \tag{B.7} $$

which may be compared with [3], Eq.(2.1) to see the relativistic modification.

It is important to note that (B.7) only applies to non-transitional orbitals in which the spin angular momentum criteria is met, i.e. orbitals 1s(+) and 2s(+). In all other orbitals, a transition occurs before this criteria is met, i.e. when ${}_e\omega_{sp}\Gamma_e = c$, then

$$ {}_e\Psi_{sp}^* = \frac{e\Gamma_e}{3} \tag{B.8} $$

APPENDIX C.

Detailed Mathematical Derivation of All Coupling Energies.

In these derivations there is a common term required. That term is, from [3], Eqs.(3.30), (3.31) and (3.32)

$$<r_e>^3=\frac{n^3n_j^3h^6}{64\pi^6Z^3e^6m_e^3} \qquad \text{(C.1)}$$

Where

$<r_e>$ is the average value of r_e around the orbital.

In (C.1) the non-relativistic value of L has been used to simplify computation. This is equivalent to discarding relativistic terms higher than the second.

C.1 Coupling Energy of the Combined Orbital Dipole and the Electron Spin Dipole.

C.1.1 Transitional Orbitals.

Via [3], Eq.(3.26), (A.27) and (B.8), this coupling energy is

$${}_eE_{SO}(OR,T)=\frac{Zehn_j}{4\pi cm_e}\left\{1-\frac{\kappa^2Z^2}{n^2}\left(\frac{2n^2}{n_\phi^{*2}}-1\right)-\frac{\gamma_pm_e}{m_p}\right\}\cdot\frac{e\Gamma_e}{3<r_e>^3} \qquad \text{(C.2)}$$

Incorporating (C.1) this reduces to

$${}_eE_{SO}(OR,T)=\frac{hR_{hy}Z^2}{n^2}\frac{\kappa^2Z^2}{n^2}\left\{1-\frac{\kappa^2Z^2}{n^2}\left(\frac{2n^2}{n_\phi^{*2}}-1\right)-\frac{\gamma_pm_e}{m_p}\right\}\cdot\frac{2\pi m_e^2cn\Gamma_e}{3hm_0n_j^2} \qquad \text{(C.3)}$$

C.1.2 Non-Transitional Orbitals.

Via [3], Eq.(3.26), (A.27) and (B.7), this coupling energy is

$${}_eE_{SO}(OR,NT)=\frac{\dfrac{Zehn_j}{4\pi cm_e}\left\{1-\dfrac{\kappa^2Z^2}{n^2}\left(\dfrac{2n^2}{n_\phi^{*2}}-1\right)-\dfrac{\gamma_pm_e}{m_p}\right\}}{\left(1+\dfrac{9_en_{sp}^2h^2}{20\pi^2c^2\Gamma_e^2m_e^2}\right)^{1/2}}\cdot\frac{eh\,{}_en_{sp}}{2\pi cm_e<r_e>^3} \qquad \text{(C.4)}$$

Incorporating (C.1) this reduces to

$$ {}_{e}E_{SO}(OR,NT)=\frac{\dfrac{hR_{hy}Z^2}{n^2}\dfrac{\kappa^2Z^2}{n^2}\left\{1-\dfrac{\kappa^2Z^2}{n^2}\left(\dfrac{2n^2}{n_\phi^{*2}}-1\right)-\dfrac{\gamma_p m_e}{m_p}\right\}}{\left(1+\dfrac{9\,{}_{e}n_{sp}^2h^2}{20\pi^2c^2\Gamma_e^2m_e^2}\right)^{1/2}}\cdot\frac{m_e}{m_0}\frac{n\,{}_{e}n_{sp}}{n_j^2} \quad (C.5) $$

C.2 Coupling Energy of the Electron and Proton Spin Dipoles.

C.2.1 Transitional Orbitals.

Via [3], Eq.(3.26), [3], Eq.(2.3) and (B.8), this coupling energy is

$$ {}_{e,p}E_{SS}(T)=-\frac{Zeh\,{}_{p}n_{sp}\gamma_p\delta_p}{2\pi cm_p}\cdot\frac{e\Gamma_e}{3<r_e>^3} \quad (C.6) $$

Incorporating (C.1) this reduces to

$$ {}_{e,p}E_{ss}(OR,T)=\frac{hR_{hy}Z^2}{n^2}\frac{\kappa^2Z^2}{n^2}\frac{2m_e^2\gamma_p\delta_p}{m_0m_p}\cdot\frac{2\pi m_ec\Gamma_e}{3eh\,{}_{e}n_{sp}}\frac{n\,{}_{e}n_{sp}\,{}_{p}n_{sp}}{n_\phi^*n_j^2} \quad (C.7) $$

C.2.2 Non-Transitional Orbitals.

Via [3], Eq.(3.26), [3], Eq.(2.3) and (B.7), this coupling energy is

$$ {}_{e,p}E_{ss}(OR,NT)=-\frac{Zeh\,{}_{p}n_{sp}\gamma_p\delta_p}{2\pi cm_p}\frac{eh\,{}_{e}n_{sp}}{2\pi cm_e\left(1+\dfrac{9\,{}_{e}n_{sp}^2h^2}{20\pi^2c^2\Gamma_e^2m_e^2}\right)^{1/2}}\cdot\frac{1}{<r_e>^3} \quad (C.8) $$

Incorporating (C.1) this reduces to

$$ {}_{e,p}E_{ss}(OR,NT)=-\frac{hR_{hy}Z^2}{n^2}\frac{\kappa^2Z^2}{n^2}\frac{2m_e^2\gamma_p\delta_p}{m_0m_p}\frac{n\,{}_{e}n_{sp}\,{}_{p}n_{sp}}{n_\phi^*n_j^2\left(1+\dfrac{9\,{}_{e}n_{sp}^2h^2}{20\pi^2c^2\Gamma_e^2m_e^2}\right)^{1/2}} \quad (C.9) $$

Expressions (C.3), (C.5), (C.7) and (C.9) are used in the main text to provide total orbital energy for the two orbital categories.

APPENDIX D.

The Spin Matter Wave Radius - Relationship with the Primary, Azimuthal and Electron Spin Quantum Numbers.

The spin matter wave radius of the electron, Γ_e, appears twice in the total orbital energy equations for both the non-transitional, (2.9), and the transitional, (2.10), orbitals. In view of the optimisation of this parameter to provide concurrence with the data in [6], it is accordingly necessary to show that it is not merely a means of adjustment, and that there is a proper relationship between it and the quantum numbers controlling the orbitals. This has been effected by tabulating the optimised values of Γ_e as in Table D.1 below.

Γ_e for Spin Up Orbitals, ($_en_{sp}$ = +1/2)					
n	**s(+)**	**p(+)**	**d(+)**	**f(+)**	**g(+)**
1	6.1705E-11				
2	1.1761E-10	3.2680E-10			
3	1.8188E-10	5.9430E-10	1.1070E-09		
4	2.8182E-10	9.1400E-10	1.7035E-09	2.6315E-09	
5	4.0241E-10	1.2790E-09	2.3800E-09	3.6780E-09	5.1500E-09
6	5.4573E-10	1.6853E-09	3.1300E-09	4.8350E-09	6.7650E-09
7	7.1470E-10	2.1230E-09	3.9420E-09	6.0860E-09	8.5170E-09
8	9.1937E-10	2.6005E-09	4.8200E-09	7.4400E-09	1.0410E-08

Table C.1A - Γ_e for Spin Up Orbitals.

Γ_e for Spin Down Orbitals, ($_en_{sp}$ = -1/2)					
n	**s(+)**	**p(+)**	**d(+)**	**f(+)**	**g(+)**
1					
2		1.2193E-10			
3		2.2720E-10	6.1270E-10		
4		3.4865E-10	9.4350E-10	1.7210E-09	
5		4.8510E-10	1.3170E-09	2.4050E-09	3.6950E-09
6		6.3560E-10	1.7310E-09	3.1600E-09	4.8550E-09
7		7.9710E-10	2.1773E-09	3.9770E-09	6.1150E-09
8		9.7010E-10	2.6600E-09	4.8600E-09	7.4700E-09

Table C.1B - Γ_e for Spin Down Orbitals.

These tables have been graphed and are shown in Figs. C.1 and C.2 respectively.

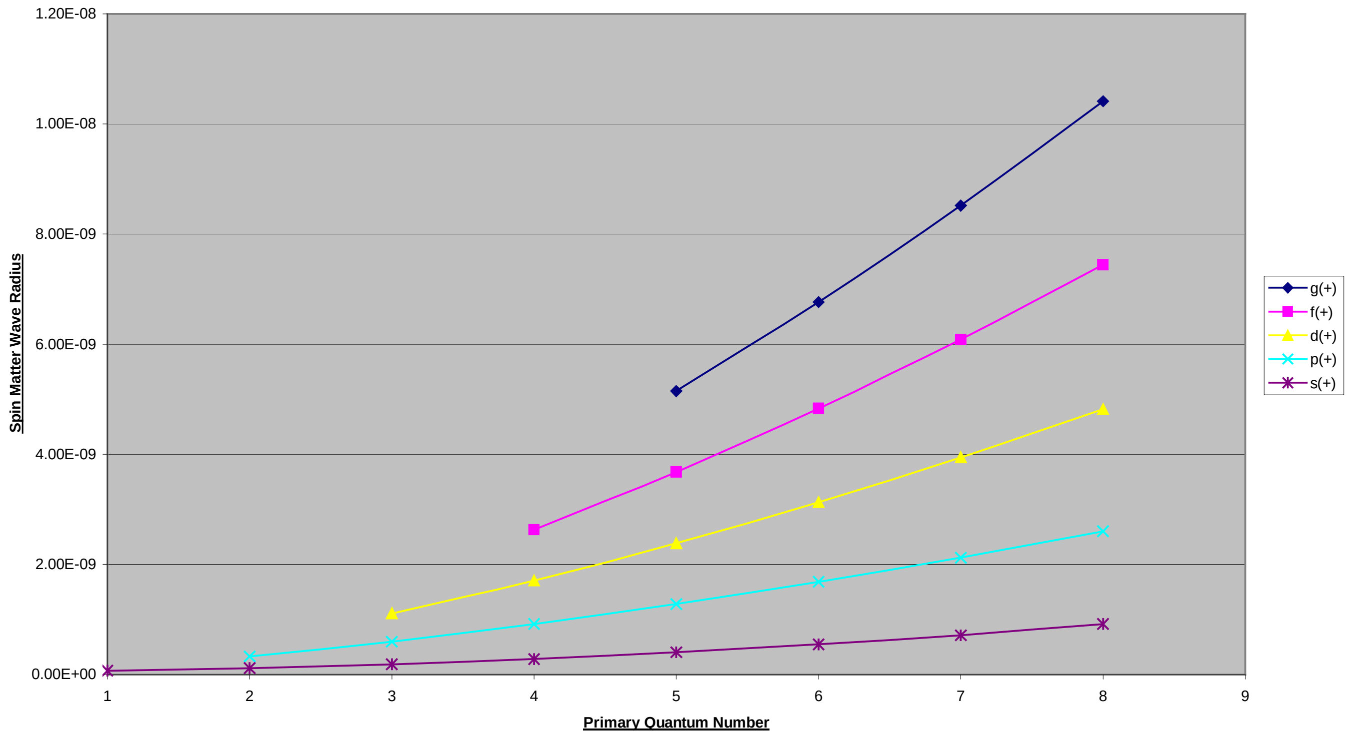

Fig. C.1 - Spin Matter Wave Radius for ϕ^*_e = 0.5 to 4.5 Spin Up.

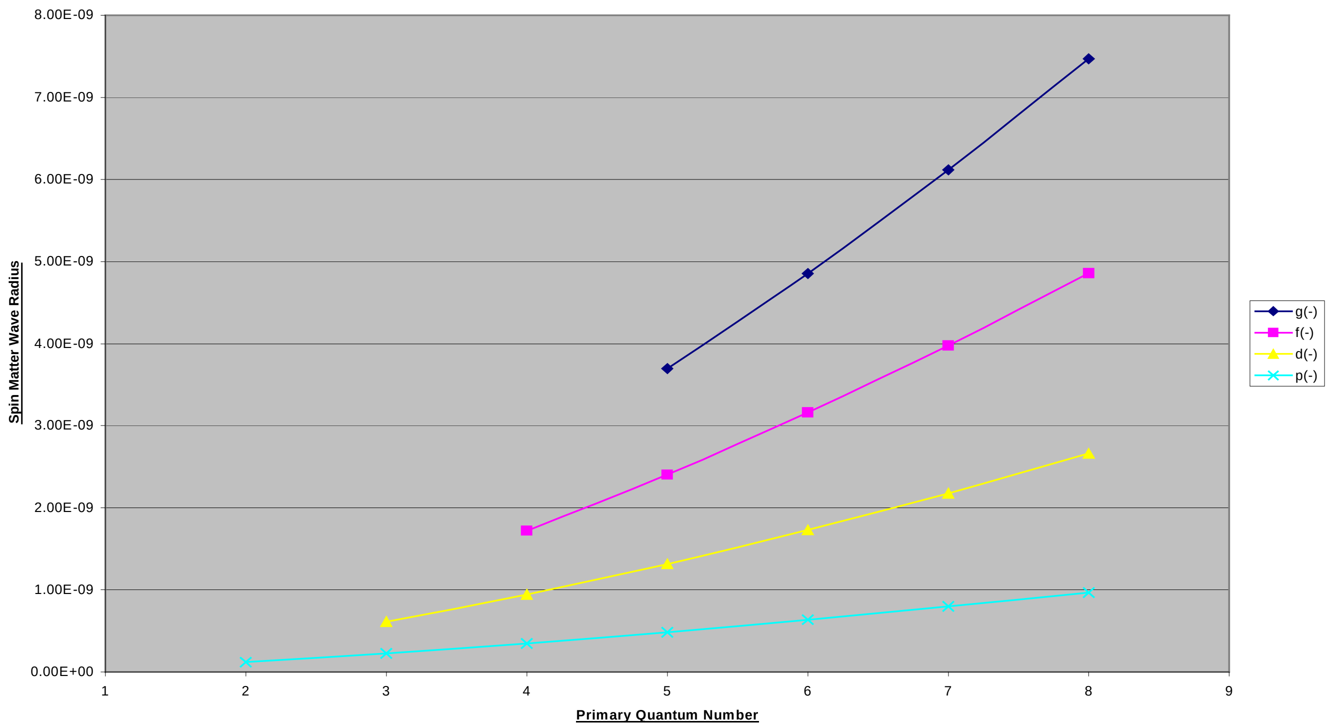

Fig. C.1 - Spin Matter Wave Radius for ϕ^*_e = 1.5 to 4.5 Spin Down.

The relationship to the quantum numbers is very clear, and precise curve fitting shows that all these relationships are of the form

$$\Gamma_e = an^3 + bn^2 + cn + d \qquad \text{(D.1)}$$

with the values of the coefficients as shown in the following table. This table shows the coefficients in bold and, via the algorithm, the calculated values of Γ_e for comparison with the actual values in Table C.1.

Spin Matter Wave Radius Calculated from Algorithm

n	a	b	c	d	Γ_e	Orbit
	3.80657750E-13	**7.40598350E-12**	**2.81434980E-11**	**2.59882470E-11**		
1	3.80657750E-13	7.40598350E-12	2.81434980E-11	2.59882470E-11	6.19183863E-11	s(+)
2	3.04526200E-12	2.96239340E-11	5.62869960E-11	2.59882470E-11	1.14944439E-10	
3	1.02777593E-11	6.66538515E-11	8.44304940E-11	2.59882470E-11	1.87350352E-10	
4	2.43620960E-11	1.18495736E-10	1.12573992E-10	2.59882470E-11	2.81420071E-10	
5	4.75822188E-11	1.85149588E-10	1.40717490E-10	2.59882470E-11	3.99437543E-10	
6	8.22220740E-11	2.66615406E-10	1.68860988E-10	2.59882470E-11	5.43686715E-10	
7	1.30565608E-10	3.62893192E-10	1.97004486E-10	2.59882470E-11	7.16451533E-10	
8	1.94896768E-10	4.73982944E-10	2.25147984E-10	2.59882470E-11	9.20015943E-10	
n	**a**	**b**	**c**	**d**	**Γ_e**	**Orbit**
	-8.21084420E-13	**3.28004100E-11**	**1.19674810E-10**	**-3.72710790E-11**		
2	-6.56867536E-12	1.31201640E-10	2.39349620E-10	-3.72710790E-11	3.26711506E-10	p(+)
3	-2.21692793E-11	2.95203690E-10	3.59024430E-10	-3.72710790E-11	5.94787762E-10	
4	-5.25494029E-11	5.24806560E-10	4.78699240E-10	-3.72710790E-11	9.13685318E-10	
5	-1.02635553E-10	8.20010250E-10	5.98374050E-10	-3.72710790E-11	1.27847767E-09	
6	-1.77354235E-10	1.18081476E-09	7.18048860E-10	-3.72710790E-11	1.68423831E-09	
7	-2.81631956E-10	1.60722009E-09	8.37723670E-10	-3.72710790E-11	2.12604072E-09	
8	-4.20395223E-10	2.09922624E-09	9.57398480E-10	-3.72710790E-11	2.59895842E-09	
n	**a**	**b**	**c**	**d**	**Γ_e**	**Orbit**
	-2.39329110E-13	**1.03407960E-11**	**5.80477440E-11**	**-3.36059110E-11**		
2	-1.91463288E-12	4.13631840E-11	1.16095488E-10	-3.36059110E-11	1.21938128E-10	p(-)
3	-6.46188597E-12	9.30671640E-11	1.74143232E-10	-3.36059110E-11	2.27142599E-10	
4	-1.53170630E-11	1.65452736E-10	2.32190976E-10	-3.36059110E-11	3.48720738E-10	
5	-2.99161388E-11	2.58519900E-10	2.90238720E-10	-3.36059110E-11	4.85236570E-10	
6	-5.16950878E-11	3.72268656E-10	3.48286464E-10	-3.36059110E-11	6.35254121E-10	
7	-8.20898847E-11	5.06699004E-10	4.06334208E-10	-3.36059110E-11	7.97337416E-10	
8	-1.22536504E-10	6.61810944E-10	4.64381952E-10	-3.36059110E-11	9.70050481E-10	
n	**a**	**b**	**c**	**d**	**Γ_e**	**Orbit**
	-1.10285730E-12	**5.30026620E-11**	**2.66395880E-10**	**-1.39536060E-10**		
3	-2.97771471E-11	4.77023958E-10	7.99187640E-10	-1.39536060E-10	1.10689839E-09	d(+)
4	-7.05828672E-11	8.48042592E-10	1.06558352E-09	-1.39536060E-10	1.70350718E-09	
5	-1.37857163E-10	1.32506655E-09	1.33197940E-09	-1.39536060E-10	2.37965273E-09	
6	-2.38217177E-10	1.90809583E-09	1.59837528E-09	-1.39536060E-10	3.12871788E-09	
7	-3.78280054E-10	2.59713044E-09	1.86477116E-09	-1.39536060E-10	3.94408548E-09	
8	-5.64662938E-10	3.39217037E-09	2.13116704E-09	-1.39536060E-10	4.81913841E-09	
n	**a**	**b**	**c**	**d**	**Γ_e**	**Orbit**
	-5.67118900E-13	**2.81129680E-11**	**1.55141630E-10**	**-9.04620570E-11**		
3	-1.53122103E-11	2.53016712E-10	4.65424890E-10	-9.04620570E-11	6.12667335E-10	d(-)
4	-3.62956096E-11	4.49807488E-10	6.20566520E-10	-9.04620570E-11	9.43616341E-10	
5	-7.08898625E-11	7.02824200E-10	7.75708150E-10	-9.04620570E-11	1.31718043E-09	
6	-1.22497682E-10	1.01206685E-09	9.30849780E-10	-9.04620570E-11	1.72995689E-09	
7	-1.94521783E-10	1.37753543E-09	1.08599141E-09	-9.04620570E-11	2.17854300E-09	
8	-2.90364877E-10	1.79922995E-09	1.24113304E-09	-9.04620570E-11	2.65953606E-09	

Table C.2 - Spin Matter Wave Radius Algorithm Coefficients and Calculation of it from the Algorithm for {s(+) to d(-)}.

n	a	b	c	d	Γ_e	Orbit
	-8.24218750E-13	**6.54617190E-11**	**5.08777340E-10**	**-3.98406250E-10**		
4	-5.27500000E-11	1.04738750E-09	2.03510936E-09	-3.98406250E-10	2.63134061E-09	f(+)
5	-1.03027344E-10	1.63654298E-09	2.54388670E-09	-3.98406250E-10	3.67899608E-09	
6	-1.78031250E-10	2.35662188E-09	3.05266404E-09	-3.98406250E-10	4.83284842E-09	
7	-2.82707031E-10	3.20762423E-09	3.56144138E-09	-3.98406250E-10	6.08795233E-09	
8	-4.22000000E-10	4.18955002E-09	4.07021872E-09	-3.98406250E-10	7.43936249E-09	
n	a	b	c	d	Γ_e	Orbit
	-5.18229170E-13	**4.21765630E-11**	**3.36611980E-10**	**-2.67187500E-10**		
4	-3.31666669E-11	6.74825008E-10	1.34644792E-09	-2.67187500E-10	1.72091876E-09	f(-)
5	-6.47786463E-11	1.05441408E-09	1.68305990E-09	-2.67187500E-10	2.40550783E-09	
6	-1.11937501E-10	1.51835627E-09	2.01967188E-09	-2.67187500E-10	3.15890315E-09	
7	-1.77752605E-10	2.06665159E-09	2.35628386E-09	-2.67187500E-10	3.97799534E-09	
8	-2.65333335E-10	2.69930003E-09	2.69289584E-09	-2.67187500E-10	4.85967504E-09	
n	a	b	c	d	Γ_e	Orbit
		6.94086650E-11	**8.50881730E-10**	**-8.39508200E-10**		
5		1.73521663E-09	4.25440865E-09	-8.39508200E-10	5.15011708E-09	g(+)
6		2.49871194E-09	5.10529038E-09	-8.39508200E-10	6.76449412E-09	
7		3.40102459E-09	5.95617211E-09	-8.39508200E-10	8.51768850E-09	
8		4.44215456E-09	6.80705384E-09	-8.39508200E-10	1.04097002E-08	
n	a	b	c	d	Γ_e	Orbit
		4.88641690E-11	6.23272830E-10	-6.43114750E-10		
5		1.22160423E-09	3.11636415E-09	-6.43114750E-10	3.69485363E-09	g(-)
6		1.75911008E-09	3.73963698E-09	-6.43114750E-10	4.85563231E-09	
7		2.39434428E-09	4.36290981E-09	-6.43114750E-10	6.11413934E-09	
8		3.12730682E-09	4.98618264E-09	-6.43114750E-10	7.47037471E-09	

Table C.2 - Spin Matter Wave Radius Algorithm Coefficients and Calculation of it from the Algorithm for {f(+) to g(-)}.

REFERENCES.

[1] P.G.Bass, *Resurrection of the Bohr/Sommerfeld Theory of Atomic Structure - [1] - Basic Orbits Including Relativistic Mass Increase Effects.* www.relativitydomains.com.

[2] P.G.Bass, *Resurrection of the Bohr/Sommerfeld Theory of Atomic Structure - [2] - Incorporation of the Mechanical Aspects of Electron Spin.* www.relativitydomains.com.

[3] P.G.Bass, *Resurrection of the Bohr/Sommerfeld Theory of Atomic Structure - [3] - Magnetic Dipole Coupling.* www.relativitydomains.com.

[4] P.G.Bass, *Resurrection of the Bohr/Sommerfeld Theory of Atomic Structure - [4] - The Addition of Small Relativistic Correction Terms Incorporating the Lamb Shift.* www.relativitydomains.com.

[5] P.G.Bass, *Resurrection of the Bohr/Sommerfeld Theory of Atomic Structure - [5] - The Hyperfine Structure.* www.relativitydomains.com.

[6] Charlotte E.Moore, [Edited by Jean W.Gallagher], *Tables of Spectra of Hydrogen, Carbon, Nitrogen and Oxygen Atoms and Ions,* CRC Press, 1993.

[7] J.D.Garcia and J.E.Mack, *Energy Levels and Line Tables for One Electron Atomic Spectra*, Journal of the Optical Society of America, Vol.55, No.6, pg. 654 - 685.