RESURRECTION OF THE BOHR/SOMMERFELD THEORY OF ATOMIC STRUCTURE

[5]

THE HYPERFINE STRUCTURE

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ABSTRACT

The resurrection of the Bohr/Sommerfeld theory of atomic structure is herein continued with the incorporation of the hyperfine structure.

1 Introduction.

The hyperfine structure of hydrogen is specifically exemplified by the so called 21 cm line. This 1420MHz emission of hydrogen was predicted in 1944 by the Dutch astronomer Dr. Hendrik van de Hulst. It was first observed in inter-stellar hydrogen by H.Ewen and E.M. Purcell at Harvard in 1951, and shortly afterwards by observers elsewhere. Observations of this spectral line of hydrogen were subsequently instrumental in revealing the spiral nature of the Milky Way galaxy.
The 21 cm line in inter-stellar hydrogen results from an electron transition between a hyperfine orbital and the ground state orbital in the first orbit shell.

The background for the development of the hyperfine structure of hydrogen in the resurrected theory, is presented in the preliminary discussion in the next Section. The remainder of the Section then provides a detailed derivation of it, in which all hyperfine orbitals and the transitions between them are identified, and the mathematical representation given.
Because the 21cm line is an emission resulting from inter-stellar hydrogen, the environment within which this development is carried out is an inter-stellar one.

In Appendices A and B, the resulting spectra are presented in both tabular, and spreadsheet form for download, as is the resulting amendments to the fine structure spreadsheet initially provided in [2]. In Appendix C, the transition type table, initially presented in [9], is extended to incorporate the hyperfine orbitals, and to show how they thereby extend the inter-orbit transition Selection Rules.
A parameter will only be defined in this paper if it has not previously been so in [1], [2] or [3], with which familiarity is assumed.

2 The Hyperfine Structure.

2.1 Preliminary Discussion.

In establishing the characteristics of hyperfine orbitals and transitions in the resurrected theory, in order for these characteristics to be sufficient and complete, it will be necessary to answer a number of pertinent questions as follows.

(i) What is the quantum number status of the hyperfine orbitals?

(ii) What are the Selection Rules applicable to intra-shell hyperfine transitions?

(iii) Where does the photon momentum come from in a hyperfine transition?

(iv) What is the photon emission mechanism in a hyperfine transition?

(v) What is the energy release in a hyperfine transition?

All of these questions will be answered in the remainder of this paper.

In the resurrected theory, the hyperfine orbitals in all orbit shells, are completely defined by spin magnetic dipole coupling between the electron and the proton, both of which can spin in either direction. It will be shown that hyperfine intra-shell orbital transitions, involve a spin reversal of just the electron. Now, for this to be effected in the ground state orbit shell, the electron must exist in one orbital in a spin down state with ${}_e n_{sp} = -1/2$. With $n^*_\phi = +1/2$ in this shell, it would then mean that in the orbital concerned$n_j = 0$, which is strictly prohibited. It is therefore proposed that in these particular ground state hyperfine orbitals, n^*_ϕ must be increased by unity. This ensures that n_j remains a good quantum number. Note that this condition is really no different to the relationship that exists between the$n$s(+) and np(-) orbitals et al via the Dead Zones as demonstrated in [2]. The fact that as a consequence in some hyperfine orbitals in the ground state orbit shell, n^*_ϕ is then greater than n, is of no consequence because n_j is still equal to n in these orbitals, so that they retain precisely the same geometric characteristics. Furthermore, it will be shown that it is this proposed azimuthal momentum variation that provides the momentum for the outgoing hyperfine photon emission. Also, it will be seen, when the mathematical representation is considered, that it is this variation that leads to the correct relationship.

Also, it will become clear when energy levels are calculated, that the hyperfine spectra are very dependent upon δ_p, the proton spin magnetisation constant of proportionality. Because this factor is purported to be produced by exactly the same phenomenon as γ_p, the primary restriction on δ_p is that it must be positive and of the same order of magnitude as γ_p.

Finally, prior to derivation of the mathematical representation of hyperfine orbital energy levels and spectral emissions, it is necessary to determine the configuration of all hyperfine orbitals and manner in which transitions between them are effected.

2.2 The Hyperfine Orbitals.

With the hyperfine orbitals included, there are theoretically $4n$ possible orbitals in the ground state shell, and ($4n$ - 2) in every other orbit shell. Using the results of the discussion in the previous Section, these orbitals are shown and characterised for the first three orbit shells in the following table, which is an extension of [1], Table 3.1, (listed in order of increasing energy).

$\boldsymbol{n}$	n^*_ϕ	$_e\boldsymbol{n}_{sp}$	$\boldsymbol{n}_j$	$_p\boldsymbol{n}_{sp}$	$\boldsymbol{n}_f$	**Term**	**Orbital Type**
1	½	+½	1	+½	1½	s(+)	Normal, (Ground State)
	1½	-½		-½	½	s_{h1}(-)	Hyperfine Triplet
	1½	-½		+½	1½	s_{h2}(-)	
	½	+½		-½	½	s_{h3}(+)	
2	1½	-½	1	-½	½	p(-)	Normal
	1½	-½		+½	1½	p_h(-)	Hyperfine
	½	+½		+½	1½	s(+)	Normal
	½	+½		-½	½	s_h(+)	Hyperfine
	1½	+½	2	+½	2½	p(+)	Normal
	1½	+½		-½	1½	p_h(+)	Hyperfine
3	1½	-½	1	-½	½	p(-)	Normal
	1½	-½		+½	1½	p_h(-)	Hyperfine
	½	+½		+½	1½	s(+)	Normal
	½	+½		-½	½	s_h(+)	Hyperfine
	2½	-½	2	-½	1½	d(-)	Normal
	2½	-½		+½	2½	d_h(-)	Hyperfine
	1½	+½		+½	2½	p(+)	Normal
	1½	+½		-½	1½	p_h(+)	Hyperfine
	2½	+½	3	+½	3½	d(+)	Normal
	2½	+½		-½	2½	d_h(+)	Hyperfine

Table 2.1 - Normal Plus Hyperfine Orbital Compliment for Shells 1 to 3.

From this table it is seen that for every "normal" orbital there is one hyperfine orbital, except for the ground state orbit shell, for which there is a hyperfine triplet. The term sequence adopted for the hyperfine orbitals is the subscript h# with the spin designator, (+ or -), according to the direction of electron spin. It is also clear that in the ground state orbit shell, for hyperfine orbitals in which $_en_{sp} = -1/2$, n^*_ϕ has, as proposed above, been increased by unity so ensuring that $n_j = n$ in these orbitals. Also note that, (as in the quantum mechanics version), the proton spin quantum number, $_pn_{sp}$, has been added to the inner quantum number, n_j, to produce an overall quantum number for the atom, n_f. This will provide a modification to the Selection Rules for inter-shell transitions, as shown in Section 2.3.4.
Note that the contents of Table 2.1 satisfactorily answers question (i) in Section 2.1.2 above.

2.3 Hyperfine Transitions.

2.3.1 Pre-Amble.

Prior to determination of the Selection Rule extensions to include hyperfine transitions, it is necessary to ascertain what phenomena affect such transitions and the order of precedence in which they take effect.

Because the ground state set of orbitals are inherently stable, any electron transition between them can only be initiated by some external stimulus extant within the inter-stellar environment. Consequently this increases to three, the number of phenomena affecting transitions. They are:-

(i) The spin angular momentum criteria.

(ii) The spin induction mechanism.

(iii) The external stimulus.

Now, the order of precedence with which these phenomena take effect can only be determined by analysing the spectral signature for each of their six possible order combinations, and comparing the results with the known correct signature. Having performed this analysis for hydrogen, the correct order is in fact as shown above. Any other order does not produce the correct spectral signature.
The application of these phenomena in the order listed to set the Selection Rules, and thus govern the manner of electron transitions, is to some extent orbit shell and orbital quadrant dependent.
In all orbit shells other than the ground state and the 2^{nd}, the spin angular momentum criteria governs exclusively the manner in which all electron transitions are initiated. Then, subsequent to such an electron transition, the spin induction mechanism and the external stimulus act in a secondary capacity to re-align particle spins according to the orbital quadrant into which the electron makes the transition. In quandrants where the spin induction mechanism and the external stimulus are complementary, both particle spins can be re-aligned. In quadrants where they are in opposition, only the electron's spin is certain to be re-aligned. The spin direction of the proton, because of its much higher mass, may not be changed before the electron spin angular momentum criteria causes a further electron inter-shell transition.
In the 2^{nd} orbit shell, the 2s(+) and the $2s_h(+)$ orbitals are meta-stable, i.e. the spin angular momentum criteria has been met, and therefore the second of the above phenomena becomes predominantly effective in re-aligning both electron and proton spins as the electron passes through the Dead Zones into the 2p(-) orbital. In this latter orbital the spin angular momentum criteria once again takes precedence and initiates an inter-shell electron transition to the ground state as has been previously described in [1].
In the ground state orbit shell the situation is again different because not only has the spin angular momentum criteria been met in all orbitals, but they are all circular orbitals and so spin induction is not present. Therefore the only phenomenon affecting electron transitions is the external stimulus.
The above dissertation now permits the determination of the Selection Rules for both intra and inter-orbit transitions, and the permitted transitions themselves.

2.3.2 **Intra-Orbit Shell Hyperfine Transitions.**

Theoretically, taking into account energy levels, the total number of potential transition combinations within any given orbit shell, is given by

$$\begin{aligned} \overset{n=1}{T_{intra}} &= \sum_{k=1}^{4n-1}(4n-k) \\ \overset{n>1}{T_{intra}} &= \sum_{k=3}^{4n-1}(4n-k) \end{aligned} \tag{2.1}$$

However, it will be seen that out of this total number of combinations, there is only one intra-orbit hyperfine transition possible.
The Selection Rules that govern inter-shell transitions do not apply here and it is necessary to develop a new set, which is effected taking into account the dissertation of the previous Section, and by using empirical results as follows.

The external stimulus governing the ground state hyperfine transitions must be such that it only causes the single transition resulting in the 21cm line. Furthermore, to cause this emission its effect must be to result in a spin reversal of either the electron or proton, but not both, (because of their opposite polarity charge). Lastly, of course the outgoing photon must take one quanta of momentum with it. From Table 2.1, it is clear that the only transition that satisfies all of these conditions is a 1s$_{h2}$(-) $\Rightarrow$ 1s(+) transition. Note that this transition incurs a spin reversal of just the electron, and a reduction in n_ϕ^* of unity. This latter effect obviously answers question (iii) in Section 2.1.2 above.
The above conditions define the Selection Rules which govern hyperfine transitions, and which may be stated as follows

$$\begin{aligned} \Delta n_\phi^* &= -1 \\ \Delta_e n_{sp} &= +1 \\ \Delta_p n_{sp} &= 0 \end{aligned} \tag{2.2}$$

These Rules answer question (ii) in Section 2.1.2 above.
Note that these results are sufficient in themselves to govern all intra-shell transitions without the necessity of considering n_f.
As a result of these Rules, the following tables list all intra-orbit transition combinations for the first two orbit shells together with all pertinent characteristics governing permissibility or otherwise. Note that these include the transitions of electrons through the Dead Zones between spin-up and spin-down elliptic orbitals as delineated in [1].

(i) Ground State Orbit Shell, Intra-Orbital Transition Combinations.

Transition			**Allowed**	**Reason**	**λ Emission**
#	**From**	**To**			
1	s$_{h1}$(-)	s(+)	No	$\Delta_p n_{sp} = +1$	-
2	s$_{h2}$(-)		Yes		21.1 cm.
3	s$_{h3}$(+)		No	$\Delta n_\phi^* = 0$; $\Delta_e n_{sp} = 0$; $\Delta_p n_{sp} = +1$	-
4	s$_{h2}$(-)	s$_{h1}$(-)	No	$\Delta n_\phi^* = 0$; $\Delta_e n_{sp} = 0$; $\Delta_p n_{sp} = -1$	-
5	s$_{h3}$(+)		No	$\Delta n_\phi^* = +1$; $\Delta_e n_{sp} = -1$	-
6	s$_{h3}$(+)	s$_{h2}$(-)	No	$\Delta n_\phi^* = +1$	-

Table 2.2 - Ground State Intra-Orbital Transition Combinations.

(ii) 2^{nd} Orbit Shell Intra-Orbital Transition Combinations.

Transition #	From	To	Allowed	Reason	λ Emission
1	p_h(-)	p(-)	Yes	Spin Induction Alignment	None - Minor Orbit Change
2	s(+)		Yes	Via Dead Zone	None - Minor Orbit Change
3	s_h(+)		Yes	Spin Induction Alignment Via Dead Zone	None - Minor Orbit Change
4	p(+)		No	$\Delta n^*_\phi = 0$; $\Delta_e n_{sp}$ = -1; $\Delta_p n_{sp}$ = -1	-
5	p_h(+)		No	$\Delta n^*_\phi = 0$; $\Delta_e n_{sp}$ = -1	-
6	s(+)	p_h(-)	No	$\Delta n^*_\phi = +1$; $\Delta_e n_{sp}$ = -1	-
7	s_h(+)			$\Delta n^*_\phi = +1$; $\Delta_e n_{sp}$ = -1; $\Delta_p n_{sp}$ = -1	
8	p(+)			$\Delta n^*_\phi = 0$; $\Delta_e n_{sp}$ = -1	
9	p_h(+)			$\Delta n^*_\phi = 0$; $\Delta_e n_{sp}$ = -1; $\Delta_p n_{sp} = +1$	
10	s_h(+)	s(+)	Yes	Spin Induction Alignment	None - Minor Orbit Change
11	p(+)		No	$\Delta_e n_{sp} = 0$	
12	p_h(+)		No	$\Delta_e n_{sp} = 0$; $\Delta_p n_{sp} = +1$	
13	p(+)	s_h(+)	No	$\Delta_e n_{sp} = 0$, $\Delta_p n_{sp}$ = -1	-
14	p_h(+)			$\Delta_e n_{sp} = 0$	
15	p_h(+)	p(+)	No	$\Delta n^*_\phi = 0$; $\Delta_e n_{sp} = 0$; $\Delta_p n_{sp} = +1$	-

Table 2.3 - Orbit Shell 2 Intra-Orbital Transition Combinations.

Note to Table 2.3.

1. Orbit shell 2 is "unstable" and electrons in all orbitals of this shell will make an inter-shell transition to the ground state. The only exceptions to this are transition numbers 2 and 3. The 2s(+) and 2s_h(+) orbitals are meta-stable and electrons in these orbitals must make a Dead Zone transition into the 2p(-) orbital before making the inter-orbit transition to the ground state. Also, allowed transition #1, incurring no emission line, but just a minor orbit change, may not occur before an inter-orbit shell transition to the ground state is initiated by the electron spin angular momentum criteria.

(iii) 3^{rd} and Higher Orbit Shell Intra-Orbital Transition Combinations.

Analysis of the intra-orbital transition combinations in all other orbit shells produces results identical to those of the 2^{nd}. These orbitals are also "unstable" and a similar note to that above for Table 2.3 applies, (apart from the reference to the meta-stable orbitals).

From the above tables it is clear that the only hyperfine transition that exists for hydrogen, is in the ground state orbit shell to produce the 21.1 cm emission line.

2.3.3 Intra-Orbital Hyperfine Transition Mechanisms.

Once an electron is captured in the $1s_{h2}$(-) orbital, it would be in a stable orbit and could only make the 21.1cm hyperfine transition via the influence of the external stimulus. In order to cause the electron in this orbital to release a photon emission and make the transition, it is proposed that the external stimulus would have to be such as to cause the magnitude of the electron spin rate to increase. Its matter wave radius, Γ_e, would consequently reduce so as to maintain the spin angular momentum quantum criteria. In this way $|_e\omega_{sp}|\Gamma_e$ would track up line 2 in [2], Fig. 5.1 until it reached the point at which $|_e\omega_{sp}|\Gamma_e = c$ and, subsequently, via exactly the same mechanism as in an inter-orbit transition, a photon emission would be initiated and the hyperfine transition effected.
This proposed mechanism provides the answer to question (iv) in Section 2.1.2 above.
There are two mechanisms by which an electron can enter the $1s_{h2}$(-) orbital. The first is via normal inter-shell transitions. The second is explained as follows.
Electrons that enter the other two ground state hyperfine orbitals, $1s_{h1}$(-) and $1s_{h3}(+)$, are also affected by the external stimulus. In the same way that it causes the spin-down magnitude of electron spin to increase, it would also cause the spin-up spin rate of the proton to increase. For the $1s_{h3}(+)$ orbital, if the spin of the proton was reversed due to this effect the electron would thereby move into the 1s(+) ground state orbital with no photon emission, (n^*_ϕ is the same in both orbitals), and the small energy difference would be accommodated by a minor orbital geometry change.
In the case of the electron in the $1s_{h1}$(-) orbital, exactly the same effect as above would cause it to move into the $1s_{h2}$(-) orbital, again with no photon emission. From there it would immediately make the 21.1cm hyperfine transition to the 1s(+) ground state. Note however, that this particular "transition", $1s_{h1}$(-) $\Rightarrow$ $1s_{h2}$(-), would involve a very small energy absorption from the external stimulus.
Accepting this scenario results in additions, and the modification of appropriate entries, in Table 2.2.

2.3.4 Inter-Orbit Shell Transitions.

The number of theoretical transition combinations between any two orbit shells, including the hyperfine orbitals, is given by

$$T_{inter} \begin{array}{l} = 4\left(4n_{(m)} - 2\right) \quad \textit{to the ground state.} \\ = \left(4n_{(m)} - 2\right)\left(4n_{(n)} - 2\right) \quad \textit{between all others.} \end{array} \tag{2.3}$$

where

$n_{(m)}$ and $n_{(n)}$ are the principle quantum numbers for the orbit shells in question.

These transitions will be governed by the existing Selection Rules, viz

$$\Delta n^*_\phi = \pm 1 \quad and \quad \Delta n_j = \ 0,\ \pm 1 \tag{2.4}$$

and two new rules as follows

(i)

$$\Delta n_f = 0,\ \pm 1 \tag{2.5}$$

This latter rule simply states that the maximum angular momentum quanta lost or gained by the atom in any transition cannot be greater than unity.

(ii) As stated above, once an electron transition has been initiated by the electron spin angular momentum criteria, the spin induction mechanism and the external stimulus will act to re-align particle spins according to the orbital quadrant receiving the electron. Consequently, if the electron is received into a spin-up quadrant, the proton spin cannot be re-aligned to a spin-down mode. This rule applies when the state of the electron spin change is 0 or +1. The rule can be stated thus: −

In the receiving quadrant if

$$_{e}n_{sp} = +1/2 \; and \; \Delta_{e}n_{sp} = 0, +1 \; and \; \Delta_{p}n_{sp} = -1 \tag{2.6}$$

the transition is "Not Permitted".

Appendix C contains the complete transition combination complex, in generalised form for inter-shell transitions, and shows in detail the reasons for permissability or otherwise.
All the above rules, for both intra and inter-orbit transitions, are utilised in Appendix A to generate the final spectral signature of hydrogen.

2.4 Mathematical Representation of the Hyperfine Orbitals and Transition Energy Levels.

To finalise the development of the hyperfine structure of hydrogen in the resurrected theory, its mathematical representation is required. However, there is no need to perform any further mathematical analysis to derive the energy levels of the hyperfine orbitals, because the necessary mathematical representation is already contained within the final relationship for electron orbital energy developed in [2], {[2], Eq. (4.2)}. For convenience that relationship is repeated here.

$$_{e}E_{or} = -\frac{hR_{hy}Z^2}{n^2}\left[1 + \frac{\kappa^2 Z^2}{n^2}\left\{\begin{array}{l} \dfrac{n}{n_\phi^*} - \dfrac{m_e}{m_0}\left(1 - \dfrac{\gamma_p m_e}{m_p}\right)\dfrac{n\,_{e}n_{sp}}{n_j n_\phi^*} \\ \\ +2\dfrac{m_e^2}{m_0 m_p}\gamma_p\;\delta_p\dfrac{n\,_{e}n_{sp}\,_{p}n_{sp}}{n_\phi^* n_j^2} - \frac{3}{4} - f\left(\Gamma_e\right) - f\left(r_p\right)\end{array}\right\}\right] \tag{2.7}$$

The Lamb Shift related terms, $f(\Gamma_e)$ and $f(r_p)$, do not figure in the hyperfine spectra and therefore have not been expanded out here.
The ground state orbital proper is given exactly by (2.7), {with $n = n_j$ =1 and $_{e}n_{sp} =_{p} n_{sp} = n_\phi^*$ = +½, and specifically designated $_{e}E_{or}\left(^{+}_{+}\right)$(1s)}.
Of particular interest is the relationship obtained for the 1s$_{h2}$(-) orbital in which $n = n_j$ =1, $_{p}n_{sp}$= +½, $_{e}n_{sp}$ = -½ and n_ϕ^* is accordingly increased by unity. This relationship, in general terms is

$$_{e}E_{or}\left(^{-}_{+}\right)(1s) = -\frac{hR_{hy}Z^2}{n^2}\left[1 + \frac{\kappa^2 Z^2}{n^2}\left\{\begin{array}{l} \dfrac{n}{n_\phi^*+1} + \dfrac{m_e}{m_0}\left(1 - \dfrac{\gamma_p m_e}{m_p}\right)\dfrac{n\,\left|_{e}n_{sp}\right|}{n_j\left(n_\phi^*+1\right)} \\ \\ -2\dfrac{m_e^2}{m_0 m_p}\gamma_p\;\delta_p\dfrac{n\,\left|_{e}n_{sp}\right|\,_{p}n_{sp}}{\left(n_\phi^*+1\right)n_j^2} \\ \\ -\frac{3}{4} - f\left(\Gamma_e\right) - f\left(r_p\right)\end{array}\right\}\right] \tag{2.8}$$

Subtracting (2.7) from (2.8) gives the energy difference in an electron transition between these two orbitals thus

$$\Delta_e E_{or}\left({}^{-}_{+}/{}^{+}_{+}\right)(1s) = \left(\frac{hR_{hy}Z^4\kappa^2}{n^3 n^*_\phi\left(n^*_\phi+1\right)}\right)\left[1-\left(2n^*_\phi+1\right)\frac{|_e n_{sp}|}{n_j}\left\{\begin{array}{l}\left(1-\gamma_p\dfrac{m_e}{m_p}\right)\dfrac{m_e}{m_0}\\ \\ -2\dfrac{m_e^2}{m_0 m_p}\gamma_p\,\delta_p\dfrac{{}_p n_{sp}}{n_j}\end{array}\right\}\right] \tag{2.9}$$

Because this energy difference is so small, it can be simplified by assuming $m_e/m_0 \cong 1$, so that (2.9) becomes, (m_0 is the reduced mass of the electron).

$$\Delta_e E_{or}\left({}^{-}_{+}/{}^{+}_{+}\right)(1s) = \left(\frac{hR_{hy}Z^4\kappa^2}{n^3 n^*_\phi\left(n^*_\phi+1\right)}\right)\left[1-\left(2n^*_\phi+1\right)\frac{|_e n_{sp}|}{n_j}\left\{\begin{array}{l}\left(1-\gamma_p\dfrac{m_e}{m_p}\right)\\ \\ -2\dfrac{m_e}{m_p}\gamma_p\,\delta_p\dfrac{{}_p n_{sp}}{n_j}\end{array}\right\}\right] \tag{2.10}$$

Now insert the value for $|_e n_{sp}|$ and ${}_p n_{sp} = +{}^1\!/_2$ to give

$$\Delta_e E_{or}\left({}^{-}_{+}/{}^{+}_{+}\right)(1s) = \left(\frac{hR_{hy}Z^4\kappa^2}{n^3 n^*_\phi\left(n^*_\phi+1\right)}\right)\left[1-\frac{\left(2n^*_\phi+1\right)}{2n_j}\left\{1-\gamma_p\frac{m_e}{m_p}\left(1+\frac{\delta_p}{n_j}\right)\right\}\right] \tag{2.11}$$

To obtain the final relationship, insert

$$n=1, \quad n_j=1, \quad n^*_\phi = {}^1\!/_2, \quad Z=1 \tag{2.12}$$

Which then gives

$$\Delta_e E_{or}\left({}^{-}_{+}/{}^{+}_{+}\right)(1s) = \frac{4}{3}hR_{hy}\kappa^2\gamma_p\frac{m_e}{m_p}\left(1+\delta_p\right) \tag{2.13}$$

This is the energy in the ground state hyperfine transition $\{1s_{h2}(\text{-}) \Rightarrow 1s(+)\}$ that produces the 21.1cm emission line. To compare this with the quantum mechanics version in [4] and [5], insert [3], Eq. (3.8) for R_{hy} to give

$$\Delta_e E_{or}\left({}^{-}_{+}/{}^{+}_{+}\right)(1s) = \frac{8}{3}\kappa^4 c^2\gamma_p\frac{m_0 m_e}{m_p}\left(\frac{1+\delta_p}{4}\right) \tag{2.14}$$

and from this it is clear that $\delta_p \approx 3$, which clearly meets the requirement as stated earlier in Section 2.1.2. The precise value of δ_p to give the exact 21.1cm line wavelength is 3.3548035.
Eq.(2.7) is used in Appendix A, together with all the results of this Section, to generate all the hyperfine, and normal, orbital energy levels for orbit shells 1 to 3 and thereby all of the normal and hyperfine spectra via allowed transitions as previously determined.
It should be noted that the analysis above does not include allowance for the air refractive index effect. This is however, incorporated in the numerical calculations in Appendix A. Finally, note that the contents of this Section answers question (v) in Section 2.1.2 above.

3 Conclusions.

3.1 The Hyperfine Structure.

The result achieved here with regard to the prediction of the hyperfine spectra, i.e. the 21.1cm line, is in complete agreement with observation, but of course has had the benefit of the adjustment of the semi-empirical parameter δ_p to ensure this. Consequently, factors which are perhaps more significant than this numerical result, are that it has been possible to identify aspects of the physics which complete the characterisation of hyperfine emissions in the resurrected theory. These factors are listed and briefly discussed below.

One aspect of the numerical results that should be mentioned however, is spectral emission "bandwidth". It can be seen from Table A.3 that due to the extra inter-shell transitions introduced by the hyperfine orbitals, each individual emission line of the fine structure spectral signature, acquires a bandwidth. For the orbit shells considered, this ranges from 0.0003Å to 0.0035Å. This range is well within pressure variation and/or Doppler pulse broadening effects, and would not therefore be discernible in experimental results.

The factors mentioned above significant to the characterisation of the hyperfine structure are listed as follows.

(i) Only one relatively simple mathematical formulation is necessary to represent the complete spectral signature of hydrogen in the resurrected Bohr/Sommerfeld theory, i.e. (2.7) or, fully expanded, [2], Eq.(4.2).

(ii) The same source of outgoing photon momentum has been identified for both intra and inter-orbit transitions. Consistency of this feature is considered a necessary attribute of the theory.

(iii) The ground state circular orbitals, with spin-down electrons, incur a unity increase in the azimuth quantum number, (and thereby associated angular momentum), to a value one half greater than the principle quantum number. This is however, quite in keeping with a similar feature in normal, (and hyperfine), elliptical orbitals via their Dead Zone transitions. It ensures that the inner quantum number remains at the same value as the principle quantum number thus maintaining the geometrics of the orbitals. It is also the source of the outgoing photon momentum. The mathematical representation has justified this process.

(iv) The nature of the external stimulus that causes hyperfine emissions, when considered in relation to the 21.1cm line ground state emission, enabled establishment of the special Selection Rules for these emissions, and in relation to inter-orbit transitions, the Selection Rule unique to the resurrected theory concerning electron/proton spin induction alignment.

(v) The same mechanism for the initiation of a photon emission has been proposed for both intra and inter-orbit transitions. This assumes that the external stimulus responsible for intra-orbital transitions is one that causes the magnitude of electron spin-down and proton spin-up to increase. Allowing for energy absorption as well as emission, this leads to the possibility that zero emission transitions into the $1s_{h2}$(-) orbital from the $1s_{h1}$(-) orbital, and into the 1s(+) orbital from the $1s_{h3}$(+) orbital, could occur.

Finally, it should be noted that in the literature it is stated that the hyperfine emission of hydrogen, the 21.1cm line, does not appear in laboratory experiments. Clearly, in this theory of atomic structure, this would be due to the absence of the external stimulus that initiates it in inter-stellar hydrogen.

3.2 The Overall Theory.

With the inclusion of the hyperfine structure, apart from the additional analysis discussed in [2] to refine accuracy, the part of the Bohr/Sommerfeld theory of atomic structure that deals with the prediction of the wavelengths of spectral signatures, to the level of precision targeted, is now virtually complete. It only remains to develop a mathematical formulation to predict relative intensities. However, only hydrogen has been considered in detail in these series of papers, and while the majority of the content would be applicable to other outer shell single electron atoms, derivation of their hyperfine structure, and the finer details of their fine structure, would require further refinement of the final orbital energy relationship to take account of their multi-particle nuclei, and possibly, electron inter-action and electron shielding of the central nuclear charge.

The resurrected theory as it stands with the completion of this paper, has predicted the complete spectral signature of hydrogen with excellent accuracy and precision and without any anomalous assumptions or approximations. In addition it provides a sound physics explanation of all aspects of the spectral signature including a cause of electron and proton spin, which leads naturally to a mechanism for the initiation of a photon release as the spin angular momentum quantum criteria is neared. In the case where this criteria is met, in the ground state orbit shell, the means by which this is achieved, the variability of the electron spin matter wave radius, has also been shown to be a major contributor to the Lamb Shift. Consequently, although these factors are all speculative, and not measurable, and may therefore be viewed with some contention, they clearly have not been introduced merely to provide an explanation for some isolated unusual feature of the spectral signature, but form an integrated complementary set of physical aspects that ensures that the overall theory is a complete one.

Appendix A

The Hyperfine Addition to the Hydrogen Fine Structure Emission Spectra for Orbit Shells 1 to 3 to Orbit Shells 1 and 2.

This Appendix presents calculated emission/absorption spectra for the first $3 \rightarrow 2$ orbit shells of hydrogen. They include the effects of relativistic mass correction, magnetic dipole coupling, the Lamb Shift and hyperfine variations. The spectra are calculated using the formula

$$\lambda_{(n)(m)} = \frac{hc}{\left(E_{or(m)} - E_{or(n)}\right)} \tag{A.1}$$

together with the Selection Rules.
For spectra in which the wavelength is greater than 2000Å, (A.1) is divided by Π_{air}, the refractive index of air. In (A.1), $E_{or(\#)}$ is given by (2.7) within which n_j is given by [1], Eq.(3.20), ${}_e n_{sp}$ and ${}_p n_{sp} = \pm 1/2$, and n_ϕ^* is increased by unity in the ground state orbitals when ${}_e n_{sp} = -1/2$. R_{hy} is determined from the generalised relationship

$$R_{hy} = \frac{cR_\infty \left\{Zm_p + (J-Z)\, m_N\right\}}{\left\{Zm_p + (J-Z)\, m_N + m_e\right\}} \tag{A.2}$$

In this and the other relationships referred to, the values of the parameters used are as shown in the following Table A.1.

Parameter	Name	Value	Units	Ref.
h	Planck's Constant	6.6260693E-27	erg secs	[7]
c	Velocity of Light in Vacuum	2.99792458E+10	cm/sec	[7]
R_{hy}	Rydberg's Constant for Hydrogen	See (A.2)	$\sec^{-1}$	
R_∞	Rydberg's Constant for Infinite Nuclear Mass	1.09737316E+5	cm^{-1}	[7]
Z	Atomic Number	1 for Hydrogen		
J	Mass Number	1 for Hydrogen		
m_p	Proton Mass	1.67262171E-24	g $\sec^2$/cm	[7]
m_N	Neutron Mass	1.6749278E-24	g $\sec^2$/cm	[7]
m_e	Electron Mass	9.10913826E-28	g $\sec^2$/cm	[7]
e	Electron/Proton Charge	-/+ 4.8032044E-10	esu	See Note 1
γ_p	Proton magnetic moment constant of proportionality	2.79275	-	[4]
δ_p	Proton spin dipole magnetisation constant of proportionality	3.3548035	-	[1]
Π_{air}	Refractive Index of Air	See Table A.2	-	Calculated from [8]

Table A1 - Parameter Values.

Note 1:- Calculated from e = 1.60217653E-20 abcoulombs x c.

Also the following factors have been used to convert energy from ergs to cm^{-1}.

Joules = 1E-7 ergs; eV = 6.24150948E+18 Joules cm^{-1} = 8.065541E+3 eV

The values of Π_{air} used, are shown in the following table, as calculated from [8].

Wavelength	Π_{air}
3889	1.000283373820
3970	1.000282912237
4101	1.000282223812
4340	1.000281143049
4861	1.000279342657
6562	1.000276235841
9545	1.000274299185
10049	1.000274132698
10938	1.000273891534
12817	1.000273537699
18751	1.000273036604
19445	1.000273006377
21654	1.000272927044
26251	1.000272823218
37395	1.000272707134
40510	1.000272691823
46525	1.000272669382
74577	1.000272628864
75003	1.000272628349
123682	1.000272605744
190568	1.000272607289
> 200000	1.000272526299

Table A.2 - Π_{air} vs Wavelength.

The only apparent anomaly in this table is that the values for λ = 123,682Å and 190,568Å appear reversed.

The calculated orbital energy levels and transition emission spectra are shown below in Table A.3 expressed as wavelengths in Ångstroms for inter-shell transitions, and cm for intra-shell transitions.

Orbital Energy - E_{OR}		To / From					n	1			
							n^*_ϕ	0.5	1.5	1.5	0.5
ergs	cm^{-1}	n	n^*_ϕ	$_e n_{sp}$	$_p n_{sp}$	n_f	Term	s(+)	s_{h1}(-)	s_{h2}(-)	s_{h3}(+)
-2.1787E-11	0.0000		0.5	0.5	0.5	1.5	s(+)				
-2.1787E-11	0.0275	1	1.5	-0.5	-0.5	0.5	s_{h1}(-)	N.P.			
-2.1787E-11	0.0474		1.5	-0.5	0.5	1.5	s_{h2}(-)	21.1061	N.P.		
-2.1787E-11	0.0596		0.5	0.5	-0.5	0.5	s_h(+)	N.P.	N.P.	N.P.	
-5.4468E-12	82258.8815		1.5	-0.5	-0.5	0.5	p(-)	1215.6737	N.P.	N.P.	1215.6746
-5.4468E-12	82258.8840		1.5	-0.5	0.5	1.5	p_h(-)	1215.6736	N.P.	N.P.	N.P.
-5.4468E-12	82258.9168	2	0.5	0.5	0.5	1.5	s(+)	N.P.	1215.6736	1215.6739	N.P.
-5.4468E-12	82258.9243		0.5	0.5	-0.5	0.5	s_h(+)	N.P.	1215.6735	1215.6737	N.P.
-5.4467E-12	82259.2475		1.5	0.5	0.5	2.5	p(+)	1215.6683	N.P.	N.P.	N.P.
-5.4467E-12	82259.2481		1.5	0.5	-0.5	1.5	p_h(+)	1215.6683	N.P.	N.P.	1215.6691
-2.4208E-12	97492.1648		1.5	-0.5	-0.5	0.5	p(-)	1025.7230	N.P.	N.P.	1025.7236
-2.4208E-12	97492.1655		1.5	-0.5	0.5	1.5	p_h(-)	1025.7230	N.P.	N.P.	N.P.
-2.4208E-12	97492.1744		0.5	0.5	0.5	1.5	s(+)	N.P.	1025.7232	1025.7234	N.P.
-2.4208E-12	97492.1766		0.5	0.5	-0.5	0.5	s_h(+)	1025.7232	1025.7234	N.P.	N.P.
-2.4208E-12	97492.2705	3	2.5	-0.5	-0.5	1.5	d(-)	N.P.	1025.7222	1025.7224	N.P.
-2.4208E-12	97492.2706		2.5	-0.5	0.5	2.5	d_h(-)	N.P.	N.P.	1025.7224	N.P.
-2.4208E-12	97492.2717		1.5	0.5	0.5	2.5	p(+)	1025.7219	N.P.	N.P.	N.P.
-2.4208E-12	97492.2719		1.5	0.5	-0.5	1.5	p_h(+)	1025.7219	N.P.	N.P.	N.P.
-2.4208E-12	97492.3060		2.5	0.5	0.5	3.5	d(+)	N.P.	N.P.	N.P.	1025.7225
-2.4208E-12	97492.3060		2.5	0.5	-0.5	2.5	d_h(+)	N.P.	N.P.	N.P.	N.P.

Table A.3 - Hyperfine and Normal Spectral Wavelengths for Transitions from Cells 1 to 3, to Cells 1 & 2.

(Continued on Next page)

n	n	2						3								
	n^*_ϕ	**1.5**	**1.5**	**0.5**	**0.5**	**1.5**	**1.5**	**1.5**	**1.5**	**0.5**	**0.5**	**2.5**	**2.5**	**1.5**	**1.5**	**2.5**
	Term	**p(-)**	**p$_h$(-)**	**s(+)**	**s$_h$(+)**	**p(+)**	**p$_h$(+)**	**p(-)**	**p$_h$(-)**	**s(+)**	**s$_h$(+)**	**d(-)**	**d$_h$(-)**	**p(+)**	**p$_{h1}$(+)**	**d(+)**
1	**s(+)**															
	s$_{h1}$(-)															
	s$_{h2}$(-)															
	s$_h$(+)															
2	**p(-)**			**D.Z.**												
	p$_h$(-)	N.P.			**D.Z.**											
	s(+)	**D.Z.**														
	s$_h$(+)	N.P.	**D.Z.**	N.P.												
	p(+)	N.P.			N.P.											
	p$_h$(+)	N.P.			N.P.											
3	**p(-)**	N.P.	N.P.	**6562.773**	**6562.776**		N.P.			**D.Z.**						
	p$_h$(-)	N.P.	N.P.	**6562.772**	N.P.	N.P.	N.P.	N.P.			**D.Z.**					
	s(+)	**6562.753**	**6562.754**		N.P.	**6562.911**	N.P.	**D.Z.**	N.P.							
	s$_h$(+)	**6562.752**	**6562.753**	N.P.	N.P.	N.P.	**6562.910**	N.P.	**D.Z.**	N.P.						
	d(-)	**6562.712**	**6562.713**	N.P.	N.P.	**6562.870**	**6562.870**	N.P.	N.P.	N.P.	N.P.			**D.Z.**		
	d$_h$(-)	N.P.	**6562.713**	N.P.	N.P.	**6562.869**	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.			**D.Z.**	
	p(+)	N.P.	N.P.	**6562.726**	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	**D.Z.**	N.P.			
	p$_h$(+)	N.P.	N.P.	**6562.726**	**6562.730**	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	**D.Z.**	N.P.		
	d(+)	N.P.	N.P.	N.P.	N.P.	**6562.854**	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	
	d$_h$(+)	N.P.	N.P.	N.P.	N.P.	**6562.854**	**6562.854**	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.

Table A.3 - Hyperfine and Normal Spectral Wavelengths for Transitions from Cells 1 to 3, to Cells 1 & 2.

(Continued).

The spreadsheet that produced Table A.3 is available for download as:-

Hydrogen Spectra 2

Three other sheets are incorporated in this spreadsheet.

(i) The revised hydrogen fine structure orbital energy levels and emission spectra and other data, (See Appendix B).

(ii) The relevant orbital energy levels and spectral wavelengths from [6].

(iii) The differences in the values in (i) and (ii).

Note that this spreadsheet is a working document that has "evolved" as the development in this series of papers has progressed. Presentation has not therefore been aesthetically optimised.

Appendix B.

Resulting Variations to the Hydrogen Fine Structure Parameters.

In order to generate the precise value for the 21.1cm hyperfine spectral line wavelength in Table A.3, (and *hydrogenSpectra2.xls)*, it was necessary to change the value of δ_p from 3.3558912 as reported in [1] and [2], to 3.3548035. This change also reflects the inclusion of Π_{air}, the refractive index of air in the calculation of the 21.1cm line. It subsequently resulted in small variations to the fine structure vacuum emission spectra as reported in [2], which were eliminated via minor changes to the following parameters.

(i) Electron Spin Matter Wave Radius.

Orbit Shell	Γ_e	
	Reference [2]	**This Paper**
1s(+)	1.36E-11	1.3589E-11
2s(+)	1.37817E-11	1.378155E-11
3p(+)	1.33E-11	1.276E-11

Table B.1 - Revised Values of Γ_e.

(ii) Electron Spin Centrifugal Force Γ_e Expansion Factor.
This factor has been changed from 7.5% in [2] to 9.5% in this paper. This change is necessary to ensure that the value of Γ_e in the ground level orbit is below the crossover point A in [2], Fig. 5.1.

All of these changes ensure the following characteristics of the hydrogen spectra predicted by this theory are maintained.

(a) The Lamb Shift in the 2^{nd} orbit shell, 2s(+) to 2p(-) is 0.353 cm^{-1} $\equiv$ 1057.77Mc/s.

(b) The ground state hyperfine emission line wavelength is 21.106114cm.

(c) The vacuum spectra wavelengths are in perfect agreement with those in [6].

As a consequence there have been some very small changes to the air spectra predicted in [2]. The differences to those in [6] have in a few cases improved and in a few others worsened. These differences are of the order 0.0288Å worst case, {transition 8k(+) $\Rightarrow$ 7i(+)}. Full details can be reviewed in the hydrogen spectra spreadsheets available in [2] and this paper, (Appendix A).

Appendix C.

Extension of the Transition Type Table to Incorporate the Hyperfine Orbitals.

In [9], all possible electron inter-shell transitions for the "normal" orbitals, were analysed for permissability and presented as a series of tables, i.e. [9], Section 4.0, Tables (i), (ii) and (iii). Here, those tables are consolidated and extended to incorporate the hyperfine orbitals. To simplify matters, the quadrant numbers are omitted in order to make way for details concerning the status of the proton.
The nomenclature adopted for the type numbers are as those in [9], Section 4.0, Tables (i), (ii) and (iii), with the bracketted sense of $\Delta_p n_{sp}$ added to distinguish the hyperfine orbitals. In the "Status" column, "O.K". means this transition is permitted by the Selection Rules. "N.P." means the transition is not.

Note 1 in Table C.1 below.

> This transition is into a spin-up quadrant in which the spin induction mechanism and the external stimulus are complementary for the proton. It cannot therefore change proton spin from spin-up to spin-down. Hence the transition is "N.P", {Selection Rule (2.6)}

In *HydrogenSpectra2.xls*, the algorithm for Selection Rule (2.6) has been truncated to omit the status of ${}_e n_{sp}$ in the receiving orbital, because transition 6(-) does not occur between orbit shells 3 & 2 to 2 & 1.

Although not shown in the table below, it is noted for interest that the hyperfine transition 1s$_{h2}$(-) $\Rightarrow$ 1s(+), is a type 2 transition.

It is important to note that as a result of the Selection Rule extensions in this paper, transitions into some orbitals are more prevalent than into others. This will affect the distribution of spectral relative intensities.

Inter-Shell Transition From Orbital					Inter-Shell Transition To Orbital					Quantum Number Change Δ(Quantum Number)					Type	Status	"Not Permitted" Reason
n_ϕ^*	$_e\boldsymbol{n}_{sp}$	$\boldsymbol{n}_j$	$_p\boldsymbol{n}_{sp}$	$\boldsymbol{n}_f$	n_ϕ^*	$_e\boldsymbol{n}_{sp}$	$\boldsymbol{n}_j$	$_p\boldsymbol{n}_{sp}$	$\boldsymbol{n}_f$	n_ϕ^*	$_e\boldsymbol{n}_{sp}$	$\boldsymbol{n}_j$	$_p\boldsymbol{n}_{sp}$	$\boldsymbol{n}_f$			
n_ϕ^*	$+1/2$	$n_\phi^*+1/2$	$+1/2$	n_ϕ^*+1	n_ϕ^*-1	$+1/2$	$n_\phi^*-1/2$	$+1/2$	n_ϕ^*	**-1**	**0**	**-1**	**0**	**-1**	**1**	**O.K.**	
					n_ϕ^*-1	$+1/2$	$n_\phi^*-1/2$	$-1/2$	n_ϕ^*-1	**-1**	**0**	**-1**	**-1**	**-2**	**1(-)**	**N.P.**	$\Delta\boldsymbol{n}_f <$ **-1**
					n_ϕ^*-1	$-1/2$	$n_\phi^*-3/2$	$+1/2$	n_ϕ^*-1	**-1**	**-1**	**-2**	**0**	**-2**	**7**	**N.P.**	$\Delta\boldsymbol{n}_j <$ **-1** $\Delta\boldsymbol{n}_f <$ **-1**
					n_ϕ^*-1	$-1/2$	$n_\phi^*-3/2$	$-1/2$	n_ϕ^*-2	**-1**	**-1**	**-2**	**-1**	**-3**	**7(-)**	**N.P.**	$\Delta\boldsymbol{n}_j <$ **-1** $\Delta\boldsymbol{n}_f <$ **-1**
n_ϕ^*	$+1/2$	$n_\phi^*+1/2$	$-1/2$	n_ϕ^*	n_ϕ^*-1	$+1/2$	$n_\phi^*-1/2$	$+1/2$	n_ϕ^*	**-1**	**0**	**-1**	**+1**	**0**	**1(+)**	**O.K.**	
					n_ϕ^*-1	$+1/2$	$n_\phi^*-1/2$	$-1/2$	n_ϕ^*-1	**-1**	**0**	**-1**	**0**	**-1**	**1**	**O.K.**	
					n_ϕ^*-1	$-1/2$	$n_\phi^*-3/2$	$+1/2$	n_ϕ^*-1	**-1**	**-1**	**-2**	**+1**	**-1**	**7(+)**	**N.P.**	$\Delta\boldsymbol{n}_j <$ **-1**
					n_ϕ^*-1	$-1/2$	$n_\phi^*-3/2$	$-1/2$	n_ϕ^*-2	**-1**	**-1**	**-2**	**0**	**-2**	**7**	**N.P.**	$\Delta\boldsymbol{n}_j <$ **-1** $\Delta\boldsymbol{n}_f <$ **-1**
n_ϕ^*	$-1/2$	$n_\phi^*-1/2$	$+1/2$	n_ϕ^*	n_ϕ^*-1	$+1/2$	$n_\phi^*-1/2$	$+1/2$	n_ϕ^*	**-1**	**+1**	**0**	**0**	**0**	**2**	**O.K.**	
					n_ϕ^*-1	$+1/2$	$n_\phi^*-1/2$	$-1/2$	n_ϕ^*-1	**-1**	**+1**	**0**	**-1**	**-1**	**2(-)**	**N.P.**	**See Note 1 above**
					n_ϕ^*-1	$-1/2$	$n_\phi^*-3/2$	$+1/2$	n_ϕ^*-1	**-1**	**0**	**-1**	**0**	**-1**	**3**	**O.K.**	
					n_ϕ^*-1	$-1/2$	$n_\phi^*-3/2$	$-1/2$	n_ϕ^*-2	**-1**	**0**	**-1**	**-1**	**-2**	**3(-)**	**N.P.**	$\Delta\boldsymbol{n}_f <$ **-1**
n_ϕ^*	$-1/2$	$n_\phi^*-1/2$	$-1/2$	n_ϕ^*	n_ϕ^*-1	$+1/2$	$n_\phi^*-1/2$	$+1/2$	n_ϕ^*	**-1**	**+1**	**0**	**+1**	**+1**	**2(+)**	**O.K.**	
					n_ϕ^*-1	$+1/2$	$n_\phi^*-1/2$	$-1/2$	n_ϕ^*-1	**-1**	**+1**	**0**	**0**	**0**	**2**	**O.K.**	
					n_ϕ^*-1	$-1/2$	$n_\phi^*-3/2$	$+1/2$	n_ϕ^*-1	**-1**	**0**	**-1**	**+1**	**0**	**3(+)**	**O.K.**	
					n_ϕ^*-1	$-1/2$	$n_\phi^*-3/2$	$-1/2$	n_ϕ^*-2	**-1**	**0**	**-1**	**0**	**-1**	**3**	**O.K.**	

TABLE C.1. - TRANSITION TYPES INCLUDING ALL POSSIBLE NORMAL AND HYPERFINE TRANSITION COMBINATIONS.

Continued on Next Page

Inter-Shell Transition From Orbital					Inter-Shell Transition To Orbital					Quantum Number Change Δ(Quantum Number)					Type	Status	"Not Permitted" Reason
n_ϕ^*	$_e\boldsymbol{n}_{sp}$	$\boldsymbol{n}_j$	$_p\boldsymbol{n}_{sp}$	$\boldsymbol{n}_f$	n_ϕ^*	$_e\boldsymbol{n}_{sp}$	$\boldsymbol{n}_j$	$_p\boldsymbol{n}_{sp}$	$\boldsymbol{n}_f$	n_ϕ^*	$_e\boldsymbol{n}_{sp}$	$\boldsymbol{n}_j$	$_p\boldsymbol{n}_{sp}$	$\boldsymbol{n}_f$			
n_ϕ^*	$+1/2$	$n_\phi^*+1/2$	$+1/2$	n_ϕ^*+1	n_ϕ^*+1	$+1/2$	$n_\phi^*+3/2$	$+1/2$	n_ϕ^*+2	**+1**	**0**	**+1**	**0**	**+1**	**4**	**O.K.**	
					n_ϕ^*+1	$+1/2$	$n_\phi^*+3/2$	$-1/2$	n_ϕ^*+1	**+1**	**0**	**+1**	**-1**	**0**	**4(-)**	**N.P.**	**See Note 1 above**
					n_ϕ^*+1	$-1/2$	$n_\phi^*+1/2$	$+1/2$	n_ϕ^*+1	**+1**	**-1**	**0**	**0**	**0**	**5**	**O.K.**	
					n_ϕ^*+1	$-1/2$	$n_\phi^*+1/2$	$-1/2$	n_ϕ^*	**+1**	**-1**	**0**	**-1**	**-1**	**5(-)**	**O.K.**	
n_ϕ^*	$+1/2$	$n_\phi^*+1/2$	$-1/2$	n_ϕ^*	n_ϕ^*+1	$+1/2$	$n_\phi^*+3/2$	$+1/2$	n_ϕ^*+2	**+1**	**0**	**+1**	**+1**	**+2**	**4(+)**	**N.P.**	$\Delta\boldsymbol{n}_f > +1$
					n_ϕ^*+1	$+1/2$	$n_\phi^*+3/2$	$-1/2$	n_ϕ^*+1	**+1**	**0**	**+1**	**0**	**+1**	**4**	**O.K.**	
					n_ϕ^*+1	$-1/2$	$n_\phi^*+1/2$	$+1/2$	n_ϕ^*+1	**+1**	**-1**	**0**	**+1**	**+1**	**5(+)**	**O.K.**	
					n_ϕ^*+1	$-1/2$	$n_\phi^*+1/2$	$-1/2$	n_ϕ^*	**+1**	**-1**	**0**	**0**	**0**	**5**	**O.K.**	
n_ϕ^*	$-1/2$	$n_\phi^*-1/2$	$+1/2$	n_ϕ^*	n_ϕ^*+1	$+1/2$	$n_\phi^*+3/2$	$+1/2$	n_ϕ^*+2	**+1**	**+1**	**+2**	**0**	**+2**	**8**	**N.P.**	$\Delta\boldsymbol{n}_j > +1$ $\Delta\boldsymbol{n}_f > +1$
					n_ϕ^*+1	$+1/2$	$n_\phi^*+3/2$	$-1/2$	n_ϕ^*+1	**+1**	**+1**	**+2**	**-1**	**+1**	**8(-)**	**N.P.**	$\Delta\boldsymbol{n}_j > +1$
					n_ϕ^*+1	$-1/2$	$n_\phi^*+1/2$	$+1/2$	n_ϕ^*+1	**+1**	**0**	**+1**	**0**	**+1**	**6**	**O.K.**	
					n_ϕ^*+1	$-1/2$	$n_\phi^*+1/2$	$-1/2$	n_ϕ^*	**+1**	**0**	**+1**	**-1**	**0**	**6(-)**	**O.K.**	
n_ϕ^*	$-1/2$	$n_\phi^*-1/2$	$-1/2$	n_ϕ^*-1	n_ϕ^*+1	$+1/2$	$n_\phi^*+3/2$	$+1/2$	n_ϕ^*+2	**+1**	**+1**	**+2**	**+1**	**+3**	**8(+)**	**N.P.**	$\Delta\boldsymbol{n}_j > +1$ $\Delta\boldsymbol{n}_f > +1$
					n_ϕ^*+1	$+1/2$	$n_\phi^*+3/2$	$-1/2$	n_ϕ^*+1	**+1**	**+1**	**+2**	**0**	**+1**	**8**	**N.P.**	$\Delta\boldsymbol{n}_j > +1$
					n_ϕ^*+1	$-1/2$	$n_\phi^*+1/2$	$+1/2$	n_ϕ^*+1	**+1**	**0**	**+1**	**+1**	**+2**	**8(+)**	**N.P.**	$\Delta\boldsymbol{n}_f > +1$
					n_ϕ^*+1	$-1/2$	$n_\phi^*+1/2$	$-1/2$	n_ϕ^*	**+1**	**0**	**+1**	**0**	**+1**	**6**	**O.K.**	

TABLE C.1. - TRANSITION TYPES INCLUDING ALL POSSIBLE NORMAL AND HYPERFINE TRANSITION COMBINATIONS.

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