CLOSED FORMS OF THE ZETA AND ETA INFINITE SERIES VIA A MODIFIED GAMMA FUNCTION RELATIONSHIP.

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Abstract.

This paper develops the fully closed forms of the Zeta and Eta infinite series for all values of the exponents, via a modified a Gamma function relationship in which only prime numbers are involved.

Contents.

1.0 Introduction.

Subsequent to solving the Basel problem in 1734 as a rational function of the parameter π, it was Leonhard Euler's contention that the closed forms of the Zeta function for all values of the exponent, both odd and even, could all be expressed in the same terms. This short paper will show that Euler's contention may be incorrect, and that the closed forms of this infinite series are in the form of a relationship involving a modified Gamma function, in which only prime numbers are involved.

The process uses Euler's Sum - Product Formula for the Zeta function to establish a relationship between Zeta functions of different exponents, which is then easily converted to the form stated above. Closed forms for all Eta infinite series then follow via the usual Eta - Zeta relationship.
Sample calculations are shown in Appendix A, and Appendix B provides a list of coefficients pertaining to the closed form relationship.

2.0 Development of the Closed Forms.

2.1 The Zeta Function.

The generalised Zeta function for some exponent m, where $m > 1$, is

$$\zeta(m) = 1 + \frac{1}{2^m} + \frac{1}{3^m} + \frac{1}{4^m} + \frac{1}{5^m} + \cdots \tag{2.1}$$

Creating Euler's Sum - Product Formula, first by transferring multiples of $1/2^m$ gives

$$\left(1 - \frac{1}{2^m}\right)\zeta(m) = 1 + \frac{1}{3^m} + \frac{1}{5^m} + \frac{1}{7^m} + \frac{1}{9^m}\cdots \tag{2.2}$$

and then transferring multiples of $1/3^m$

$$\left(1 - \frac{1}{2^m}\right)\left(1 - \frac{1}{3^m}\right)\zeta(m) = 1 + \frac{1}{5^m} + \frac{1}{7^m} + \frac{1}{11^m} + \frac{1}{13^m}\cdots \tag{2.3}$$

and after q further iterations yields

$$\left(1 - \frac{1}{2^m}\right)\left(1 - \frac{1}{3^m}\right)\cdots\left(1 - \frac{1}{p_q^m}\right)\zeta(m) = 1 + \varepsilon_m \tag{2.4}$$

where

p_q is the q^{th} prime number.

ε_m is the sum of the remaining terms.

Similarly, for a second exponent n where $n > 1$

$$\left(1 - \frac{1}{2^n}\right)\left(1 - \frac{1}{3^n}\right)\cdots\left(1 - \frac{1}{p_q^n}\right)\zeta(n) = 1 + \varepsilon_n \tag{2.5}$$

Now because ζ(m) and ζ(n) are absolutely convergent, in (2.4) and (2.5) if q is large enough, both ε_m and ε_n will be negligibly small, and for the purposes here may be ignored. As a result (2.4) and (2.5) may then be equated thus

$$\left(1-\frac{1}{2^m}\right)\left(1-\frac{1}{3^m}\right)\cdots\left(1-\frac{1}{p_q^m}\right)\zeta(m)=\left(1-\frac{1}{2^n}\right)\left(1-\frac{1}{3^n}\right)\cdots\left(1-\frac{1}{p_q^n}\right)\zeta(n) \tag{2.6}$$

Consequently with $m \geq n$

$$\zeta(m)=\frac{\left(1-\frac{1}{2^n}\right)\left(1-\frac{1}{3^n}\right)\cdots\left(1-\frac{1}{p_q^n}\right)}{\left(1-\frac{1}{2^m}\right)\left(1-\frac{1}{3^m}\right)\cdots\left(1-\frac{1}{p_q^m}\right)}\zeta(n) \tag{2.7}$$

and this reduces to

$$\zeta(m)=\frac{(2^n-1)(3^n-1)\cdots(p_q^n-1)}{(2^m-1)(3^m-1)\cdots(p_q^m-1)}(2x3x5x\cdots p_q)^{(m-n)}\zeta(n) \tag{2.8}$$

which may be expressed as a product formula thus

$$\zeta(m)=\zeta(n)\prod_{x=2}^{q}\left\{\frac{(p_x^n-1)p_x^{(m-n)}}{(p_x^m-1)}\right\} \tag{2.9}$$

or in terms of factorials as

$$\zeta(m)=\frac{(p_q^n-1)!\{p_q^{(m-n)}\}!}{(p_q^m-1)!}\zeta(n) \tag{2.10}$$

where $q \geq 2$, (see footnote [1]), and since

$$\Gamma(t)=(t-1)! \tag{2.11}$$

Eq.(2.10) becomes in terms of Gamma functions

$$\zeta(m)=\frac{\Gamma(p_q^n)\Gamma\{p_q^{(m-n)}+1\}}{\Gamma(p_q^m)}\zeta(n) \tag{2.12}$$

and also since

$$\Gamma(t+1)=t\,\Gamma(t) \tag{2.13}$$

Eq.(2.12) finally becomes

[1] It should be noted that although it has no effect upon the results, for consistency with other papers, unity is considered to be the first prime number.

$$\zeta(m)=\frac{\Gamma\left(p_q^n\right)\Gamma\left\{p_q^{(m-n)}\right\}p_q^{(m-n)}}{\Gamma\left(p_q^m\right)}\zeta(n) \tag{2.14}$$

Eq.(2.14) is the generalised expression for the fully closed form of the Zeta function for any value of the exponent. Therefore $\zeta(m)$ for any value of m can be determined from (2.14), {or (2.9) or (2.10)} from any other exponent n. Because (2.14) is a product formula, it is fully invertible, thus

$$\zeta(n)=\frac{\Gamma\left(p_q^m\right)}{\Gamma\left(p_q^n\right)\Gamma\left\{p_q^{(m-n)}\right\}p_q^{(m-n)}}\zeta(m) \tag{2.15}$$

Which allows the closed form of a Zeta function with a low exponent to be determined from one with a higher exponent.

2.2 The Eta Function.

From (2.14) the closed form of the Eta Function can be expressed in terms of that for any Zeta function via the usual relationship, thus

$$\eta(m)=\frac{\left\{2^{(m-1)}-1\right\}}{2^{(m-1)}}\frac{\Gamma\left(p_q^n\right)\Gamma\left\{p_q^{(m-n)}\right\}p_q^{(m-n)}}{\Gamma\left(p_q^m\right)}\zeta(n) \tag{2.16}$$

or in terms of a second Eta Function

$$\eta(m)=\frac{\left\{2^{(m-1)}-1\right\}}{\left\{2^{(n-1)}-1\right\}2^{(m-n)}}\frac{\Gamma\left(p_q^n\right)\Gamma\left\{p_q^{(m-n)}\right\}p_q^{(m-n)}}{\Gamma\left(p_q^m\right)}\eta(n) \tag{2.17}$$

3.0 Discussion of Results.

3.1 Two Special Cases.

When m and n are both even or odd, for instance if $m = 4$ and $n = 2$, then

$$\zeta(4)=\frac{\pi^4}{90}=\frac{\pi^2}{15}\zeta(2) \tag{3.6}$$

In terms of (2.10) this is

$$\zeta(4)=\frac{p_q^2!\left(p_q^2-1\right)!}{\left(p_q^4-1\right)!}\zeta(2) \tag{3.7}$$

Consequently, the co-efficient of $\zeta(2)$ in (3.7) is as $q \to \infty$

$$\frac{p_q^2!\left(p_q^2-1\right)!}{\left(p_q^4-1\right)!}=\frac{\pi^2}{15} \tag{3.8}$$

which, despite the coefficient appearing to be the ratio of two natural numbers, is irrational. Both numerator and denominator in (3.8) will tend to an infinity as $q \to \infty$ and consequently these two infinities must be different, and their ratio = $\pi^2/15$. This applies to all other values of m and n where both are even and concurs with the conclusions of [1] concerning the nature of infinity. When m

and/or n are odd, there is no equivalent of (3.6) with which to effect a comparison, and so the same result cannot be confirmed.

A second special case occurs when $m = n$, (2.10) then becomes

$$\zeta(m) = \frac{\left(p_q^m - 1\right)!\left(p_q^0\right)!}{\left(p_q^m - 1\right)!}\zeta(m) \tag{3.9}$$

and the co-efficient clearly reduces to unity irrespective of the value of q. This applies to all values of the exponent.

3.2 The Euler Contention.

From (2.10) it is clear that the co-efficient of $\zeta(n)$ is the ratio of two natural numbers, and is therefore rational. If n is an even exponent it is well known that $\zeta(n)$ is a rational function of π. Therefore, if m is an odd exponent, then $\zeta(m)$ should also be a rational function of π. Consequently, if Euler's contention is correct, the coefficient in (2.9) et al should, in line with the discussion above in Section 3.1, be a rational function of π as $q \to \infty$.

However, using the coefficients of Appendix B, in which q has been restricted to a range of $2 \to 510$, show that while, for $m = 8$ and $n = 6$

$$0.986960440108943 = \frac{\pi^2}{10} \tag{3.1}$$

and for $m = 8$ and $n = 4$

$$0.977705628895996 = \frac{\pi^4}{105} \tag{3.2}$$

a similar calculation for $m = 7$ and $n = 6$ gives

$$0.991159536110684 = \frac{\pi}{3.16961350734456} \tag{3.3}$$

similarly for $m = 7$ and $n = 5$ gives

$$0.972439277838128 = \frac{\pi^2}{10.1493271878434} \tag{3.4}$$

and finally for $m = 8$ and $n = 7$

$$0.995763450939279 = \frac{\pi}{3.15495879129366} \tag{3.5}$$

Here q is large enough to show that whenever either m or n is odd, then as q increases $\zeta(m)/\zeta(n)$ does not appear to be a rational function of a power of π. While not a proof, this suggests that Euler's contention on this issue may not be correct.

3.3 The Value of π.

From (2.10), a new definition of π can be constructed by taking the ratio of the closed form values of two Zeta functions with even exponents. i.e.

$$\frac{\zeta(m)}{\zeta(n)} = \frac{\pi^{(m-n)}}{k} = \frac{\left(p_q^n - 1\right)!\left\{p_q^{(m-n)}\right\}!}{\left(p_q^m - 1\right)!} \tag{3.10}$$

so that

$$\pi = \left[\frac{k\left(p_q^n - 1\right)!\left\{p_q^{(m-n)}\right\}!}{\left(p_q^m - 1\right)!}\right]^{\frac{1}{(m-n)}} \tag{3.11}$$

and k is a natural number and clearly the inverse of the ratio of the coefficients of $\zeta(m)$ and $\zeta(n)$.

For $m = 10$ and $n = 8$

$$k = \frac{93555}{9450} = 9.9 \tag{3.12}$$

and from Appendix B the value of the factorials ratio for these exponents is 0.996929737483774 so that

$$\begin{aligned}\pi &= (9.9\ x\ 0.996929737483774)^{1/2} \\ &= 3.142592653589793\ldots\end{aligned} \tag{3.13}$$

and is accurate to at least 15 significant figures, (and only limited by the value of q used in determining the coefficients of Appendix B and the software used).

Re-writing (3.11) as

$$\pi = \left[\frac{k\left(1 - \frac{1}{p_q^n}\right)!\left\{p_q^{(m-n)}\right\}!}{\left(\frac{p_q^m}{p_q^n} - \frac{1}{p_q^n}\right)!}\right]^{\frac{1}{(m-n)}} \tag{3.14}$$

then as $q \to \infty$, (3.14) becomes

$$\pi = \left[\frac{k\left\{p_q^{(m-n)}\right\}!\left(p_q^n\right)!}{\left(p_q^m\right)!}\right]_{q\to\infty}^{\frac{1}{(m-n)}} \tag{3.15}$$

and, hypothetically, this equation gives the precise value of π to an infinite number of decimal places. Both numerator and denominator in (3.15) each go to an infinity the ratio of which is exactly π^2/k.

4.0 Conclusions.

In the form developed here, the closed forms of these infinite series do not necessitate any degree of series summations, as was the case in [2], nor a strict ladder relationship between series with adjacent exponents as in [3].

They still however, depend upon a relationship involving an infinite product, and between series with non-adjacent exponents, (although adjacent exponents can still be used), and it is not known whether these closed forms could be developed independent of these restrictions.

In performing calculations to obtain a closed form specific to any particular exponent, either the product/factorial form or the Gamma form could be used. However, it is unlikely that extensive tables of Gamma functions of the type required here are available to enable the use of the latter. Consequently, only the former method is practicable, and this means performing a great many fairly detailed computations. Hence the provision of Appendix B.

Finally, in making calculations, the nature of the results obtained are such that the rate of convergence of the coefficients, especially for low values of exponent, is very slow. However, the objective here was not to produce the fastest possible rate of convergence, but to develop a fully closed form for these series irrespective of any other attribute.

APPENDIX A

Some Calculated Examples.

Eq.(2.14) represents the formal expression of the generalised closed form of ζ(*m*), but cannot be used to perform calculations because of a lack of required Gamma function tables.
To avoid working with excessively large numbers, it is best to use (2.9) and its inverse, and determine each term inside the main bracket for all values of *q*, and then take the product as the final operation.

The infinite products of the coefficients of ζ(*n*) only converge very slowly, especially for low values of *n* and therefore *q* should be as large as possible to achieve the highest degree of precision. Also, the greatest precision for any value of *q* is obtained when *n* = *m* - 1.For instance, in determining ζ(13) from ζ(2), with *q* ranging from 2 to 510, a precision of 2.98E-5 is achieved. Determining ζ(13) from ζ(12) for the same range of *q* produces a precision of 4.66E-15. These two calculations are demonstrated below.

(i) ζ(13) from ζ(2) with *q* = 2 to 510.

Using (2.9) and the appropriate coefficient from Appendix B.

$$\begin{aligned}\zeta(13) &= \prod_{q=2}^{510}\left\{\frac{\left(p_q^2-1\right)p_q^{11}}{\left(p_q^{13}-1\right)}\right\}\frac{\pi^2}{6}\\ &= 0.608019826410395\frac{\pi^2}{6}\\ &= 1.00015255781630\end{aligned} \tag{A.1}$$

The actual value to 15 decimal places is 1.000122713347580.

(ii) ζ(13) from ζ(12) with *q* = 2 to 510.

Using (2.9) etc

$$\begin{aligned}\zeta(13) &= \prod_{q=2}^{510}\left\{\frac{\left(p_q^{12}-1\right)p_q}{\left(p_q^{13}-1\right)}\right\}\frac{1{,}415{,}168\pi^{12}}{15\,!}\\ &= 0.999876657147293\frac{1{,}415{,}168\pi^{12}}{15\,!}\\ &= 1.0001227133475847\end{aligned} \tag{A2}$$

Finally, precision is maximised, certainly for low values of the exponents, if the inverse relationship of (2.9) is used. To illustrate this

(iii) ζ(3) from ζ(2) with *q* = 2 to 510.

$$\zeta(3)=\prod_{q=2}^{510}\left\{\frac{\left(p_q^2-1\right)p_q}{\left(p_q^3-1\right)}\right\}\frac{\pi^2}{6}$$
$$=0.730784749427433\frac{\pi^2}{6} \qquad (A.3)$$
$$=1.20209272986633$$

which is accurate to +3.5826706745512E-5.

(iv) ζ(3) from ζ(4) with q = 2 to 510.

$$\zeta(3)=\prod_{q=2}^{510}\left\{\frac{\left(p_q^4-1\right)}{\left(p_q^3-1\right)p_q}\right\}\frac{\pi^4}{90}$$
$$=\frac{1}{0.900392681487698}\frac{\pi^4}{90} \qquad (A.4)$$
$$=1.2020568515282$$

which is accurate to within -5.1631800602575E-8

Appendix B

Table of Coefficients.

This Appendix lists the values of the coefficients of (2.9), {(2.10), (2.12) and (2.14)}, for the range of q from 2 to 510 and for exponents 2 to 30.

Use is very simple, i.e. to determine ζ(17) from ζ(16) from the table for m = 17 and n = 16 the coefficient is 0.99992355055061, so that

$$\zeta(17) = 0.99992355055061\,\zeta(16) \tag{B.1}$$

To determine ζ(4) from ζ(26), from the table the value of the coefficient is 0.923938416690444 so that

$$\zeta(4) = \frac{\zeta(26)}{0.923938416690444} \tag{B.2}$$

Note that in the preparation of this table, precision was limited to 15 decimal places by the software used.

Appendix B

Table of Coefficients.

Coefficient Value

m	n = 2	3	4	5	6	7	8
2	1						
3	0.730784749427433	1					
4	0.657993240127281	0.900392681487698	1				
5	0.630395275767801	0.862627848024625	0.958057374032989	1			
6	0.618488855109754	0.846335197326348	0.939962323914014	0.981112769851350	1		
7	0.613021126720208	0.838853201576124	0.931652620932133	0.972439277838128	0.991159536110684	1	
8	0.610424032641594	0.835299358832899	0.927705628895996	0.968319491128987	0.986960440108943	0.995763450939279	1
9	0.609166216242248	0.833578172942888	0.925794034182495	0.966324209045355	0.984926747199273	0.993711618882405	0.997939438272269
10	0.608549870615172	0.832734770521647	0.924857329077505	0.965346496091647	0.983930212464680	0.992706195740833	0.996929737483774
11	0.608245663018228	0.832318495281661	0.924395002751956	0.964863929662871	0.983438356234075	0.992209952489690	0.996431382929108
12	0.608094830561514	0.832112097355556	0.924165771739365	0.964624663185933	0.983194483680089	0.991963904759622	0.996184288370825
13	0.608019826410395	0.832009462275691	0.924051782496701	0.964505683628173	0.983073213667697	0.991841553101823	0.996061416158870
14	0.607982458928039	0.831958328912022	0.923994992426409	0.964446407355350	0.983012796277708	0.991780596830125	0.996000200544216
15	0.607963819389040	0.831932822716095	0.923966664568520	0.964416839337127	0.982982659050757	0.991750190799746	0.995969665149147
16	0.607954514193495	0.831920089561017	0.923952522788676	0.964402078446779	0.982967614001090	0.991735011558550	0.995954421326742
17	0.607949866414711	0.831913729577735	0.923945459222513	0.964394705645989	0.982960099267805	0.991727429798993	0.995946807310034
18	0.607947544122090	0.831910551771115	0.923941929866166	0.964391021778572	0.982956344482900	0.991723641523979	0.995943002917527
19	0.607946383505692	0.831908963592925	0.923940165993334	0.964389180685458	0.982954467947229	0.991721748250894	0.995941101589370
20	0.607945803373546	0.831908169744743	0.923939284324482	0.964388260418181	0.982953529964042	0.991720801901545	0.995940151213710
21	0.607945513366017	0.831907772900761	0.923938843579028	0.964387800377401	0.982953061067107	0.991720328822366	0.995939676121780
22	0.607945368381729	0.831907574505420	0.923938623235899	0.964387570387918	0.982952826650121	0.991720092314553	0.995939438607722
23	0.607945295896069	0.831907475316627	0.923938513074188	0.964387455403457	0.982952709452124	0.991719974071212	0.995939319861320
24	0.607945259655401	0.831907425725182	0.923938457996619	0.964387397914652	0.982952650856613	0.991719914953078	0.995939260491648
25	0.607945241535785	0.831907400930447	0.923938430458927	0.964387369171391	0.982952621560020	0.991719885395182	0.995939230808002
26	0.607945232476214	0.831907388533405	**0.923938416690444**	0.964387354800147	0.982952606912111	0.991719870616623	0.995939215966565
27	0.607945227946510	0.831907382334993	0.923938409806321	0.964387347614647	0.982952599588288	0.991719863227474	0.995939208545978
28	0.607945225681683	0.831907379235823	0.923938406364303	0.964387344021936	0.982952595926418	0.991719859532942	0.995939204835730
29	0.607945224549279	0.831907377686250	0.923938404643305	0.964387342225594	0.982952594095495	0.991719857685691	0.995939202980619
30	0.607945223983083	0.831907376911470	0.923938403782813	0.964387341327434	0.982952593180042	0.991719856762075	0.995939202053072

Appendix B

Table of Coefficients.

Coefficient Value

m	*n*						
	9	**10**	**11**	**12**	**13**	**14**	**15**
2							
3							
4							
5							
6							
7							
8							
9	1						
10	0.998988214364752	1					
11	0.998488830799424	0.999500110653816	1				
12	0.998241226036240	0.999252255113959	0.999752020497837	1			
13	0.998118100115728	0.999129004490234	0.999628708231626	0.999876657147293	1		
14	0.998056758102074	0.999067600348750	0.999567273379515	0.999815207056818	0.999938542329166	1	
15	0.998026159657016	0.999036970913267	0.999536628625047	0.999784554701194	0.999907886192326	0.999969341978996	1
16	0.998010884358918	0.999021680144194	0.999521330208455	0.999769252489967	0.999892582093445	0.999954036939509	0.999984694491283
17	0.998003254620656	0.999014042678457	0.999513688922919	0.999761609309080	0.999884937969713	0.999946392345953	0.999977049663352
18	0.997999442372776	0.999010226569490	0.999509870905367	0.999757790344505	0.999881118534039	0.999942572675529	0.999973229875824
19	0.997997537118727	0.999008319385779	0.999507962767796	0.999755881733641	0.999879209687732	0.999940663711904	0.999971320853664
20	0.997996584780711	0.999007366083228	0.999507008988459	0.999754927717729	0.999878255554134	0.999939709519661	0.999970366632169
21	0.997996108707809	0.999006889528149	0.999506532195040	0.999754450806043	0.999877778583614	0.999939232519831	0.999969889617714
22	0.997995870703332	0.999006651282618	0.999506293830353	0.999754212382228	0.999877540130394	0.999938994051948	0.999969651142524
23	0.997995751711734	0.999006532170507	0.999506174658670	0.999754093180987	0.999877420914443	0.999938874828675	0.999969531915591
24	0.997995692219486	0.999006472618001	0.999506115076377	0.999754033583918	0.999877361310022	0.999938815220591	0.999969472305683
25	0.997995662474529	0.999006442842934	0.999506085286415	0.999754003786565	0.999877331508995	0.999938785417731	0.999969442501909
26	0.997995647602457	0.999006427955780	0.999506070391830	0.999753988888280	0.999877316608878	0.999938770516694	0.999969427600415
27	0.997995640166547	0.999006420512347	0.999506062944656	0.999753981439279	0.999877309158945	0.999938763066309	0.999969420149803
28	0.997995636448638	0.999006416790664	0.999506059221127	0.999753977714802	0.999877305434025	0.999938759341155	0.999969416424533
29	0.997995634589696	0.999006414929842	0.999506057359373	0.999753975852595	0.999877303571571	0.999938757478600	0.999969414561916
30	0.997995633660234	0.999006413999445	0.999506056428500	0.999753974921495	0.999877302640365	0.999938756547316	0.999969413630621

Appendix B

Table of Coefficients.

Coefficient Value	m	n						
		16	17	18	19	20	21	22
	2							
	3							
	4							
	5							
	6							
	7							
	8							
	9							
	10							
	11							
	12							
	13							
	14							
	15							
	16	1						
	17	**0.999992355055061**	1					
	18	0.999988535209064	0.999996180124808	1				
	19	0.999986626157690	0.999994271058841	0.999998090926744	1			
	20	0.999985671921591	0.999993316815445	0.999997136679703	0.999999045751141	1		
	21	0.999985194899834	0.999992839790040	0.999996659652478	0.999998568723006	0.999999522971414	1	
	22	0.999984956420994	0.999992601309377	0.999996421170905	0.999998330240977	0.999999284489158	0.999999761517637	1
	23	0.999984837192239	0.999992482079712	0.999996301940784	0.999998211010629	0.999999165258699	0.999999642287118	0.999999880769461
	24	0.999984777581413	0.999992422468431	0.999996242329276	0.999998151399007	0.999999105647016	0.999999582675411	0.999999821157740
	25	0.999984747777183	0.999992392663974	0.999996212524707	0.999998121594380	0.999999075842358	0.999999552870739	0.999999791353061
	26	0.999984732875462	0.999992377762138	0.999996197622813	0.999998106692461	0.999999060940426	0.999999537968799	0.999999776451116
	27	0.999984725424733	0.999992370311351	0.999996190171997	0.999998099241630	0.999999053489591	0.999999530517960	0.999999769000275
	28	0.999984721699412	0.999992366586003	0.999996186446628	0.999998095516260	0.999999049764215	0.999999526792582	0.999999765274896
	29	0.999984719836767	0.999992364723343	0.999996184583967	0.999998093653583	0.999999047901542	0.999999524929908	0.999999763412223
	30	0.999984718905449	0.999992363792021	0.999996183652643	0.999998092722258	0.999999046970215	0.999999523998579	0.999999762480893

Appendix B

Table of Coefficients.

Coefficient Value	m	n							
		23	24	25	26	27	28	29	30
	2								
	3								
	4								
	5								
	6								
	7								
	8								
	9								
	10								
	11								
	12								
	13								
	14								
	15								
	16								
	17								
	18								
	19								
	20								
	21								
	22								
	23	1							
	24	0.999999940388277	1						
	25	0.999999910583596	0.999999970195324	1					
	26	0.999999895681653	0.999999955293378	0.999999985098059	1				
	27	0.999999888230809	0.999999947842534	0.999999977647215	0.999999992549161	1			
	28	0.999999884505431	0.999999944117155	0.999999973921840	0.999999988823783	0.999999996274628	1		
	29	0.999999882642756	0.999999942254479	0.999999972059163	0.999999986961113	0.999999994411952	0.999999998137334	1	
	30	0.999999881711431	0.999999941323153	0.999999971127836	0.999999986029783	0.999999993480628	0.999999997206005	0.999999999068682	1

References.

[1] P.G.Bass, *The Series Sum of Divergent Alternating Infinite Series and The Nature of Infinity*, www.relativitydomains.com.

[2] P.G.Bass. *Pseudo Closed Forms For The Eta and Zeta Infinite Series For All Values of The Exponents Both Odd and Even, www.relativitydomains.com.*

[3] P.G.Bass, *The Closed Forms of Convergent Infinite Series - 3 - Generalisation Via Fourier Series Expansion, www.relativitydomains.com.*