Determination of an Approximate Value for the Closed Form of the Harmonic Series, (Assuming Convergence).

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Abstract.

Assuming convergence, as suggested in [1], this paper, via a semi-analytic/empirical process determines an approximate closed form value for the Harmonic Series.

For best appreciation of this paper it is recommended that [1] be read first.

Contents.

1.0 Introduction.

In [1] the limiting divergent infinite series was shown to be the Unitary series, $\zeta(0)$. This result suggested that the Harmonic series was not divergent but instead convergent with a finite closed form. Using a semi – analytic/empirical process this paper determines an approximate value for it.

The investigation is however complicated by the fact that the adjacent series, $\zeta(0)$, is, as stated above, divergent and cannot therefore be used as a data point. Instead, expressing all Zeta series as a function of π, the investigation can be carried out using their denominators, so that the denominator of $\zeta(0)$, being zero, becomes a valid data point.

Thus this representative set of data points enables a graph to be established which when analysed with a professional level curve fitting process produces an algorithm from which the $\zeta(1)$ denominator can be extracted, so leading to the approximate closed form value.

2.0 Nomenclature.

The nomenclature used in this paper is as follows.

$\zeta(n)$	Represents a Zeta infinite series.
n	Represents the exponent in the Zeta infinite series.
k	Represents the denominator of $\zeta(1)$ when expressed as a function of π.

3.0 Establing the Veracity of the Base Data.

It is initially necessary to demonstrate the validity of the data points to be used. These are the closed form values of $\zeta(n)$ for n = 2 to 8 and are shown below in tabular form.

n	ζ(n) Closed Form
2	1.644934067
3	1.202056852
4	1.082323234
5	1.036927761
6	1.017343067
7	1.008349277
8	1.004007736

Table 3.1 – Closed Form Values of Zeta(n) for *n* = 2 to 8.

In Table 3.1 the closed form values for even $\zeta(n)$ were determined by Leonard Euler circa 1734 and are well documented in the mainstream literature, and also shown in [2]. The values for all odd exponents of n were determined in [3]. A log/linear graph of Table 3.1 is presented below and shows that the data chosen form a smooth curve and are therefore viable as data points in this investigation.

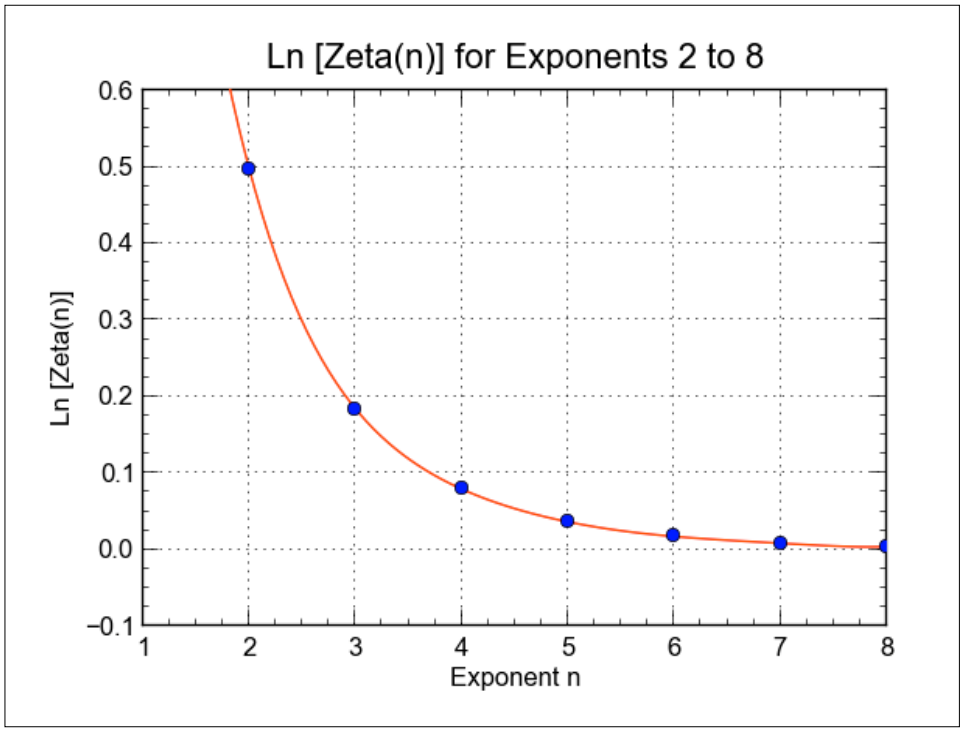

Fig. 3.1 – Closed Form Values of ζ(n) for *n* = 2 to 8.

4.0 Determination of the Closed Form Value of ζ(1).

4.1 Initial Data Points Preparation.

As stated in the Introduction, in order to use ζ(0) as a data point, all such points are expressed as a function of π and their denominators used as the base data. This is shown in tabular form thus

n	Numerator	Denominator
0	π^0	0
1	π^1	*k*
2	π^2	6
3	π^3	25.7944
4	π^4	90
5	π^5	295.1215
6	π^6	945
7	π^7	2995.2848
8	π^8	9450

Table 4.1 - ζ(*n*) Closed Form Numerators and Denominators.

The objective now is to determine a value for *k* in Table 4.1 by plotting the denominators against *n* and interpolating for *k* from the algorithm so derived. However, the area of the curve/algorithm around *n* = 1 is extremely sensitive to small variations and so it is not sufficient to simply plot the numerical values in Table 4.1. Instead it is necessary to insert a number of trial values for *k*, obtain a

curve fit algorithm for each set from which the most suitable/accurate value for *k* can be determined from an error analysis of the complete set of results.

It is well known that it takes in excess of the sum of one million terms for $\zeta(1)$ to exceed a value of 15, and therefore *k* must be somewhat less than

$$k < \frac{\pi}{15} < 0.21 \qquad (4.1)$$

Consequently, the trial values of *k* chosen for this exercise are:-

0.1, 0.0875, 0.075, 0.05, 0.01, 0.00925, 0.00875, 0.0075, 0.005, 0.001 and 0.0001.

4.2 Determination of $\zeta(1)$.

The numerical denominators in Table 4.1 were thus augmented by each of the above trial values of *k*, plotted and a curve fit algorithm produced for each set. These algorithms were each an eight order polynomial and are shown in Appendix A, together with predicted values of the $\zeta(n)$ denominator for each *n* and resulting error value. The error values were then analysed as follows.

They were initially compiled into a single table shown in Appendix B, Table B.1. In this table numbers in red are the minima while numbers in blue are the maximums. There is no clear indication in this table of which trial value of *k* is superior and so Table B.1 was re-written in Table B.2(a). Here the error values for each even exponent were ordered from minimum to maximum, and this then indicated that for a *k* of 0.0075 the error values showed consistently low results for all even exponents. To confirm this value was superior to all others, all trial values of *k* were then allotted a "Position Rating Score" from Table B.2(a) which is shown in Table B.2(b). These were then averaged for each trial value of *k* and as shown the value of 0.0075 was confirmed as superior. Attempts to refine this value by evaluating values of 0.0075 ± 0.000125 did not result in any improvement. The average points scores, including those for the refinement attempts, is shown below in graphical form, (the *x* axis is not to scale to provide clarity).

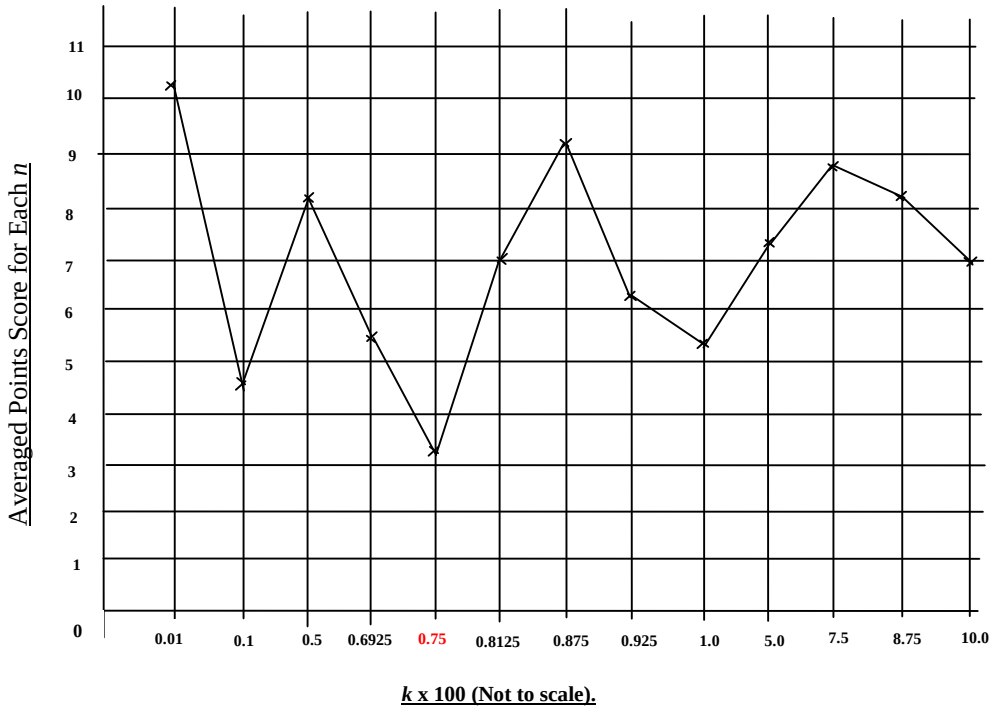

Fig. 4.1 – Points Score Graph for ζ(1) Trial Values of *k*.

With *k* selected as above, the closed form value of ζ(1) is therefore predicted to be

$$\zeta(1) \approx \frac{\pi}{0.0075} \approx 419 \qquad (4.2)$$

To further determine the likely suitability of this value it was added to those in Table 3.1. The values are tabularised in log/linear form and plotted below.

n	**Ln**[ζ(*n*)]
1	6.0403
2	0.4977
3	0.1840
4	0.0791
5	0.0363
6	0.0172
7	0.0083
8	0.0041

Table 4.2 – Ln[ζ(*n*)] vs Exponent *n*.

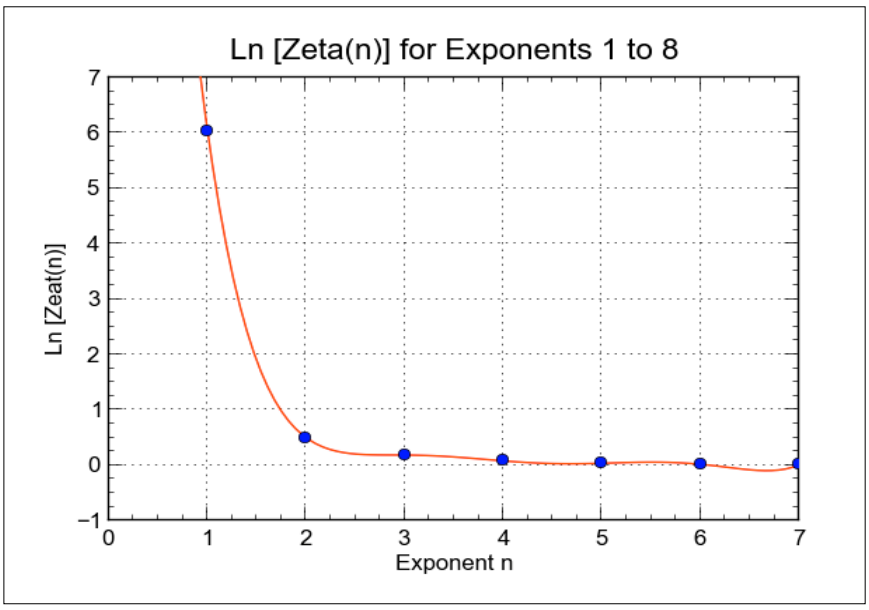

Fig 4.2 – Ln[ζ(*n*)] vs Exponent *n*.

Note that while the curve shows an excellent fit to the points, it is not perfect in that it exhibits undulations between the points. To produce a perfectly smooth curve would require a much larger number of points.

5.0 Conclusions.

While the process adopted here has provided an approximate result, it is clear from Fig. 4.1 that it is not one that could be claimed to be conclusive. There is sufficient variability in the overall point scores to indicate that the result obtained can only be described as one being "of the order of". Furthermore, it is considered that attempts at additional refinement by adding extra data points (at larger values of *n*), would most probably not provide a great deal more assurance as to the accuracy of the value derived here.

Instead, a slightly different approach can be adopted which while still being a semi-analytical/empirical method may provide better assurance in that it will employ the Euler-Mascheroni constant in the deliberations.

Appendix A.

Details of the Curve Fit Results for All Trial Values of *k*.

(Check predicted denominators against Table 4.1 for n = 0 and 2 to 8).

Name	Polynomial Regression (degree=8):		Zeta(1) Denominator = 0.1		
Equation:	a + b*x + c*x^2 + ...		n	Predicted Denominator	Error
a =	-2.2737367544E-11		0	-2.273736754432320E-11	2.273736754432320E-11
b =	-6.8633885715E+01		1	9.999999993054250E-02	
c =	1.7368278667E+02		2	5.999999999973720E+00	2.627587036840850E-11
d =	-1.7132673167E+02		3	2.579429999999690E+01	
e =	8.7965522778E+01		4	8.999999999995710E+01	4.285993782104920E-11
f =	-2.5462660417E+01		5	2.925124999999090E+02	
g =	4.2400140278E+00		6	9.449999999999450E+02	5.456968210637570E-11
h =	-3.7967220238E-01		7	2.995284699999930E+03	
i =	1.4626527778E-02		8	9.449999999999850E+03	1.455191522836690E-10
Standard Error : 0.000000000000000E+00					
Correlation Coefficient : 1.000000000000000E+00					

Name	Polynomial Regression (degree=8)		Zeta(1) Denominator = 0.0875		
Equation	a +b*x + c*x^2 + ...		n	Predicted Denominator	Error
a =	-9.0949470177E-13		0	-9.094947017729280E-13	9.094947017729280E-13
b =	-6.8733885714E+01		1	8.750000001030790E-02	
c =	1.7385457238E+02		2	5.999999999997680E+00	2.323474745935530E-12
d =	-1.7144791222E+02		3	2.579430000003470E+01	
e =	8.8011217222E+01		4	9.000000000008000E+01	8.003553375601770E-11
f =	-2.5472643055E+01		5	2.925125000002570E+02	
g =	4.2412813889E+00		6	9.450000000006620E+02	6.621121428906920E-10
h =	-3.7975900793E-01		7	2.995284700001550E+03	
i =	1.4629007936E-02		8	9.450000000003020E+03	3.026798367500310E-09
Standard Error : 0.000000000000000E+00					
Correlation Coefficient : 1.000000000000000E+00					

Name	Polynomial Regression (degree=8):		Zeta(1) Denominator = 0.075		
Equation	a + b*x + c*x^2 + ...		n	Predicted Denominator	Error
a =	-2.4101609597E-11		0	-2.410160959698260E-11	2.410160959698260E-11
b =	-6.8833885715E+01		1	7.499999992982940E-02	
c =	1.7402635810E+02		2	5.999999999979950E+00	2.005151600315000E-11
d =	-1.7156909278E+02		3	2.579430000002550E+01	
e =	8.8056911667E+01		4	9.000000000004630E+01	4.627054295269770E-11
f =	-2.5482625694E+01		5	2.925125000001330E+02	
g =	4.2425487500E+00		6	9.450000000003490E+02	3.492459654808040E-10
h =	-3.7984581349E-01		7	2.995284700000610E+03	
i =	1.4631488095E-02		8	9.450000000000790E+03	7.858034223318100E-10
Standard Error : 0.000000000000000E+00					
Correlation Coefficient : 1.000000000000000E+00					

Name	Polynomial Regression (degree=8):			Zeta(1) Denominator = 0.05			
Equation	a + b*x + c*x^2 + ...			n	Predicted Denominator		Error
a =	1.7735146685E-11			0	1.773514668457210E-11		1.773514668457210E-11
b =	-6.9033885714E+01			1	4.999999995555940E-02		
c =	1.7436992952E+02			2	6.000000000014970E+00		1.497468815614410E-11
d =	-1.7181145389E+02			3	2.579430000001420E+01		
e =	8.8148300555E+01			4	8.999999999996100E+01		3.899458533851430E-11
f =	-2.5502590972E+01			5	2.925124999998820E+02		
g =	4.2450834722E+00			6	9.449999999996830E+02		3.165041562169790E-10
h =	-3.8001942460E-01			7	2.995284699999130E+03		
i =	1.4636448413E-02			8	9.449999999997930E+03		2.066371962428090E-09
Standard Error : 0.000000000000000E+00							
Correlation Coefficient : 1.000000000000000E+00							

Name	Polynomial Regression (degree=8)			Zeta(1) Denominator = 0.01			
Equation	: a + b*x + c*x^2 + ...			n	Predicted Denominator		Error
a =	-1.1368683772E-12			0	-1.136868377216160E-12		1.136868377216160E-12
b =	-6.9353885714E+01			1	1.000000000843660E-02		
c =	1.7491964381E+02			2	5.999999999995330E+00		4.668265773943860E-12
d =	-1.7219923167E+02			3	2.579430000002740E+01		
e =	8.8294522777E+01			4	9.000000000005450E+01		5.445599526865410E-11
f =	-2.5534535417E+01			5	2.925125000001490E+02		
g =	4.2491390278E+00			6	9.450000000002840E+02		2.837623469531540E-10
h =	-3.8029720238E-01			7	2.995284700000590E+03		
i =	1.4644384921E-02			8	9.450000000000440E+03		4.365574568510060E-10
Standard Error : 0.000000000000000E+00							
Correlation Coefficient : 1.000000000000000E+00							

Name	Polynomial Regression (degree=8):			Zeta(1) Denominator = 0.00925			
Equation	: a + b*x + c*x^2 + ...			n	Predicted Denominator		Error
a =	-2.4101609597E-11			0	-2.410160959698260E-11		2.410160959698260E-11
b =	-6.9359885715E+01			1	9.249999929383730E-03		
c =	1.7492995095E+02			2	5.999999999977770E+00		2.222932948825470E-11
d =	-1.7220650250E+02			3	2.579430000000850E+01		
e =	8.8297264445E+01			4	8.999999999998280E+01		1.716671249596400E-11
f =	-2.5535134375E+01			5	2.925124999999450E+02		
g =	4.2492150695E+00			6	9.449999999999780E+02		2.182787284255030E-11
h =	-3.8030241071E-01			7	2.995284699999950E+03		
i =	1.4644533730E-02			8	9.449999999999560E+03		4.365574568510060E-10
Standard Error : 0.000000000000000E+00							
Correlation Coefficient : 1.000000000000000E+00							

Name	Polynomial Regression (degree=8):			Zeta(1) Denominator = 0.00875			
Equation a + b*x + c*x^2 + ...				n	Predicted Denominator		Error
a =	-2.4101609597E-11			0	-2.410160959698260E-11		2.410160959698260E-11
b =	-6.9363885715E+01			1	8.749999927924380E-03		
c =	1.7493682238E+02			2	5.999999999968190E+00		3.180566920946150E-11
d =	-1.7221134972E+02			3	2.579429999998190E+01		
e =	8.8299092222E+01			4	8.999999999992940E+01		7.059952622512360E-11
f =	-2.5535533681E+01			5	2.925124999998780E+02		
g =	4.2492657639E+00			6	9.450000000000040E+02		3.637978807091710E-12
h =	-3.8030588294E-01			7	2.995284700000270E+03		
i =	1.4644632937E-02			8	9.450000000001020E+03		1.018634065985680E-09
Standard Error : 0.000000000000000E+00							
Correlation Coefficient : 1.000000000000000E+00							

Name	Polynomial Regression (degree=8):			Zeta(1) Denominator = 0.0075			
Equation a + b*x + c*x^2 + ...				n	Predicted Denominator		Error
a =	-2.5011104299E-12			0	-2.501110429875550E-12		2.501110429875550E-12
b =	-6.9373885714E+01			1	7.500000008436400E-03		
c =	1.7495400095E+02			2	5.999999999990630E+00		9.372058684675720E-12
d =	-1.7222346778E+02			3	2.579430000000640E+01		
e =	8.8303661666E+01			4	8.999999999998530E+01		1.466560206608850E-11
f =	-2.5536531944E+01			5	2.925124999999880E+02		
g =	4.2493925000E+00			6	9.449999999999530E+02		4.729372449219230E-11
h =	-3.8031456349E-01			7	2.995284699999970E+03		
i =	1.4644880952E-02			8	9.449999999999740E+03		2.619344741106030E-10
Standard Error	: 0.000000000000000E+00						
Correlation Coefficient	: 1.000000000000000E+00						

Name	Polynomial Regression (degree=8):			Zeta(1) Denominator = 0.005			
Equation a + b*x + c*x^2 + ...				n	Predicted Denominator		Error
a =	1.7280399334E-11			0	1.728039933368560E-11		1.728039933368560E-11
b =	-6.9393885714E+01			1	4.999999956596290E-03		
c =	1.7498835809E+02			2	6.000000000015420E+00		1.542055372283360E-11
d =	-1.7224770389E+02			3	2.579430000002900E+01		
e =	8.8312800555E+01			4	9.000000000003870E+01		3.865352482534950E-11
f =	-2.5538528472E+01			5	2.925125000001640E+02		
g =	4.2496459722E+00			6	9.450000000004290E+02		4.292814992368220E-10
h =	-3.8033192460E-01			7	2.995284700001100E+03		
i =	1.4645376984E-02			8	9.450000000002090E+03		2.095475792884830E-09
Standard Error	: 0.000000000000000E+00						
Correlation Coefficient	: 1.000000000000000E+00						

Name	Polynomial Regression (degree=8)]:			Zeta(1) Denominator = 0.001			
Equation	: a + b*x + c*x^2 + ...			n	Predicted Denominator		Error
a =	-1.8189894035E-12			0	-1.818989403545850E-12		1.818989403545850E-12
b =	-6.9425885714E+01			1	1.000000006782590E-03		
c =	1.7504332952E+02			2	5.999999999987980E+00		1.201705401854270E-11
d =	-1.7228648167E+02			3	2.579430000000750E+01		
e =	8.8327422777E+01			4	9.000000000000850E+01		8.526512829121200E-12
f =	-2.5541722917E+01			5	2.925125000000790E+02		
g =	4.2500515278E+00			6	9.450000000002150E+02		2.146407496184110E-10
h =	-3.8035970238E-01			7	2.995284700000510E+03		
i =	1.4646170635E-02			8	9.450000000000790E+03		7.858034223318100E-10
Standard Error	: 0.000000000000000E+00						
Correlation Coefficient	: 1.000000000000000E+00						

Name	Polynomial Regression (degree=8):			Zeta(1) Denominator = 0.0001			
Equation	: a + b*x + c*x^2 + ...			n	Predicted Denominator		Error
a =	-2.4101609597E-11			0	-2.410160959698260E-11		2.410160959698260E-11
b =	-6.9433085715E+01			1	9.999992917673720E-05		
c =	1.7505569810E+02			2	5.999999999977570E+00		2.242828145426760E-11
d =	-1.7229520667E+02			3	2.579430000002100E+01		
e =	8.8330712778E+01			4	9.000000000004080E+01		4.081357474206020E-11
f =	-2.5542441667E+01			5	2.925125000001400E+02		
g =	4.2501427778E+00			6	9.450000000004150E+02		4.147295840084550E-10
h =	-3.8036595238E-01			7	2.995284700000760E+03		
i =	1.4646349206E-02			8	9.450000000001370E+03		1.367880031466480E-09
Standard Error	: 0.000000000000000E+00						
Correlation Coefficient	: 1.000000000000000E+00						

Refinement Attempts							
Name:	**Polynomial Regression (degree=8)**			**Zeta(1) Denominator = 0.006975**			
Equation:	**a + b*x + c*x^2 + ...**						
Parameters:				**n**	**Predicted Denominator**		**Error**
a =	-1.14E-12			0	-1.136868377216160E-12		1.136868377216160E-12
b =	-6.94E+01			1	6.975000008927270E-03		
c =	1.75E+02			2	5.999999999992100E+00		7.898570686393210E-12
d =	-1.72E+02			3	2.579430000000710E+01		
e =	8.83E+01			4	8.999999999996430E+01		3.569766704458740E-11
f =	-2.55E+01			5	2.925124999998660E+02		
g =	4.25E+00			6	9.449999999995600E+02		4.401954356580970E-10
h =	-3.80E-01			7	2.995284699998900E+03		
i =	1.46E-02			8	9.449999999997150E+03		2.852175384759900E-09
Standard Error::	0.0000000000000000E+00						
Correlation Coefficient (r):	1.00000000000000E+00						

Name:	**Polynomial Regression (degree=8)**			**Zeta(1) Denominator = 0.008125**			
Equation:	**a + b*x + c*x^2 + ...**						
Parameters:				**n**	**Predicted Denominator**		**Error**
a =	1.86E-11			0	1.864464138634500E-11		1.864464138634500E-11
b =	-6.94E+01			1	8.124999956214330E-03		
c =	1.75E+02			2	6.000000000017810E+00		1.780886549340720E-11
d =	-1.72E+02			3	2.579430000002780E+01		
e =	8.83E+01			4	9.000000000001610E+01		1.614353095646950E-11
f =	-2.55E+01			5	2.925125000000650E+02		
g =	4.25E+00			6	9.450000000001200E+02		1.200533006340270E-10
h =	-3.80E-01			7	2.995284700000250E+03		
i =	1.46E-02			8	9.450000000000290E+03		2.910383045673370E-10
Standard Error::	0.0000000000000000E+00						
Correlation Coefficient (r):	1.00000000000000E+00						

Table A.1 – Algorithm Results for all *k* at all Exponents.

Appendix B.

Details of the Error Table Analysis for All Trial Values of *k*.

Exp. n / ζ1) Den.	0	2	4	6	8	Average Error	Overall Maximum Error Min/Max	Overall Minimum Error Min/Max
0.1	2.2737367544E-11	2.6275870368E-11	4.2859937821E-11	5.4569682106E-11	1.4551915228E-10	5.8392402025E-11	1.4551915228E-10	2.2737367544E-11
0.0875	9.0949470177E-13	2.3234747459E-12	8.0035533756E-11	6.6211214289E-10	3.0267983675E-09	7.5443580272E-10	3.0267983675E-09	9.0949470177E-13
0.075	2.4101609597E-11	2.0051516003E-11	4.6270542953E-11	3.4924596548E-10	7.8580342233E-10	2.4509461127E-10	7.8580342233E-10	2.0051516003E-11
0.05	1.7735146685E-11	1.4974688156E-11	3.8994585339E-11	3.1650415622E-10	2.0663719624E-09	4.9091610776E-10	2.0663719624E-09	1.4974688156E-11
0.01	1.1368683772E-12	4.6682657739E-12	5.4455995269E-11	2.8376234695E-10	4.3655745685E-10	1.5611618664E-10	4.3655745685E-10	1.1368683772E-12
0.00925	2.4101609597E-11	2.2229329488E-11	1.7166712496E-11	2.1827872843E-11	4.3655745685E-10	1.0437659625E-10	4.3655745685E-10	1.7166712496E-11
0.00875	2.4101609597E-11	3.1805669209E-11	7.0599526225E-11	3.6379788071E-12	1.0186340660E-09	2.2975576996E-10	1.0186340660E-09	3.6379788071E-12
0.008125	1.1368683772E-12	7.8985706864E-12	3.5697667045E-11	4.4019543566E-10	2.8521753848E-09	6.6742078531E-10	2.8521753848E-09	1.1368683772E-12
0.0075	2.5011104299E-12	9.3720586847E-12	1.4665602066E-11	4.7293724492E-11	2.6193447411E-10	6.7153393957E-11	2.6193447411E-10	2.5011104299E-12
0.006975	1.8644641386E-11	1.7808865493E-11	1.6143530956E-11	1.2005330063E-10	2.9103830457E-10	9.2737728608E-11	2.9103830457E-10	1.6143530956E-11
0.005	1.7280399334E-11	1.5420553723E-11	3.8653524825E-11	4.2928149924E-10	2.0954757929E-09	5.1922235400E-10	2.0954757929E-09	1.5420553723E-11
0.001	1.8189894035E-12	1.2017054019E-11	8.5265128291E-12	2.1464074962E-10	7.8580342233E-10	2.0456134564E-10	7.8580342233E-10	1.8189894035E-12
0.0001	2.4101609597E-11	2.2428281454E-11	4.0813574742E-11	4.1472958401E-10	1.3678800315E-09	3.7399061625E-10	1.3678800315E-09	2.2428281454E-11

Table B1 – Algorithm Error Values.

Zeta(1) Denominator Values Ordered According to Smallest to Largest Errors									
Trial Zeta(1) Denominator Values	Exponent 0	Trial Zeta(1) Denominator Values	Exponent 2	Trial Zeta(1) Denominator Values	Exponent 4	Trial Zeta(1) Denominator Values	Exponent 6	Trial Zeta(1) Denominator Values	Exponent 8
0.0875	9.0949470177E-13	0.0875	2.3234747459E-12	0.001	8.5265128291E-12	0.00875	3.6379788071E-12	0.1	1.4551915228E-10
0.01	1.1368683772E-12	0.01	4.6682657739E-12	0.0075	1.4665602066E-11	0.00925	2.1827872843E-11	0.0075	2.6193447411E-10
0.008125	1.1368683772E-12	0.008125	7.8985706864E-12	0.006975	1.6143530956E-11	0.0075	4.7293724492E-11	0.006975	2.9103830457E-10
0.001	1.8189894035E-12	0.0075	9.3720586847E-12	0.00925	1.7166712496E-11	0.1	5.4569682106E-11	0.01	4.3655745685E-10
0.0075	2.5011104299E-12	0.001	1.2017054019E-11	0.008125	3.5697667045E-11	0.006975	1.2005330063E-10	0.00925	4.3655745685E-10
0.005	1.7280399334E-11	0.05	1.4974688156E-11	0.005	3.8653524825E-11	0.001	2.1464074962E-10	0.075	7.8580342233E-10
0.05	1.7735146685E-11	0.005	1.5420553723E-11	0.05	3.8994585339E-11	0.01	2.8376234695E-10	0.001	7.8580342233E-10
0.006975	1.8644641386E-11	0.006975	1.7808865493E-11	0.0001	4.0813574742E-11	0.05	3.1650415622E-10	0.00875	1.0186340660E-09
0.1	2.2737367544E-11	0.075	2.0051516003E-11	0.1	4.2859937821E-11	0.075	3.4924596548E-10	0.0001	1.3678800315E-09
0.075	2.4101609597E-11	0.00925	2.2229329488E-11	0.075	4.6270542953E-11	0.0001	4.1472958401E-10	0.05	2.0663719624E-09
0.00925	2.4101609597E-11	0.0001	2.2428281454E-11	0.01	5.4455995269E-11	0.005	4.2928149924E-10	0.005	2.0954757929E-09
0.00875	2.4101609597E-11	0.1	2.6275870368E-11	0.00875	7.0599526225E-11	0.008125	4.4019543566E-10	0.008125	2.8521753848E-09
0.0001	2.4101609597E-11	0.00875	3.1805669209E-11	0.0875	8.0035533756E-11	0.0875	6.6211214289E-10	0.0875	3.0267983675E-09

Table B2(a) – Algorithm Error Values Ordered from Smallest to Largest for *n* = 0, 2, 4, 6 & 8 for ζ(1) Trial Denominators.

Position Rating Score for Even *n*						
Trial Zeta(1) Denominator Values	0	2	4	6	8	Average
0.1	9	12	9	4	1	7.00
0.0875	1	1	13	13	13	8.20
0.075	10	9	10	9	6	8.80
0.05	7	6	7	8	10	7.60
0.01	2	2	11	7	4	5.20
0.00925	11	10	4	2	5	6.40
0.00875	12	13	12	1	8	9.20
0.008125	3	3	5	12	12	7.00
0.0075	5	4	2	3	2	3.20
0.006975	8	8	3	5	3	5.40
0.005	6	7	6	11	11	8.20
0.001	4	5	1	6	7	4.60
0.0001	13	11	8	10	9	10.20

Table B2(b) – Zeta(1) Trial Values Rating Scores for Even Exponents *n*.

References.

[1] P.G.Bass, *Determination of the Limiting Divergent Infinite Series and a Review of the Divergency of the Harmonic Series*, www.relativitydomains.com.

[2] P.G.Bass, *The Closed Forms of Convergent Infinite Series – 5 – Fourier Series Expansion*, www.relativitydomains.com.

[3] P.G.Bass, *Closed Forms of the Zeta and Eta Infinite Series Via a Modified Gamma Function Relationship*, www.relativitydomains.com.