Generation of the Gravitational Acceleration Potential and The Time Dilatation Effect.

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Abstract

Following the presentation of the new theory of gravitation in [1], this paper presents a mathematical derivation for the generation by the gravitational source, of the two central parameters in that theory, gravitational Acceleration Potential and the time dilatation effect.

1 Introduction.

In the General Theory of Relativity, the cause of gravitational motion is purported to be a curvature of space-time, due to the presence of a gravitational source in the form of ponderable matter. The degree of curvature is stated to be proportional to the mass of the gravitational source, which then determines the strength of the gravitational effect. However, the precise mechanism that causes this space-time distortion by the source is not described. Clearly, the characteristics of the continuum would have to be such that they were capable of this distortion, but again these are not addressed. Only the geometry of the curvature is described by means of a mathematical concept - the geodesics. It is considered that for a full understanding of gravity, a complete physical interpretation should be available, as well as a mathematical one.

In this series of papers, the proposed gravitational theory of the Relativistic Space-Time Domain $\mathbf{D_1}$, an alternative to that in the General Theory, the same requisites are of course just as applicable. In this Domain, $\mathbf{D_1}$, space-time is not curved, but completely linear along all four axes, yet, as shown in [1], still exhibits identical gravitational characteristics to those in the General Theory. The cause of gravitational motion in $\mathbf{D_1}$, as shown in [1], is the Acceleration Potential extant within the Domain. This, in turn, is augmented by the time dilatation effect produced by the gravitational source in concert with the velocity of the gravitating body. Therefore, these two parameters, the Acceleration Potential and time dilatation, are central to the gravitational theory of $\mathbf{D_1}$, and a complete understanding of it can only be achieved if the manner in which these parameters are generated by the source is known.

It is the purpose of this paper to provide both a mathematical development for, and a physical interpretation of, the mechanism generating both time dilatation and the Acceleration Potential of $\mathbf{D_1}$, and thereby attempt to enhance this understanding. Because both of these parameters are generated by the same mechanism, it is not significant which is used to develop the characteristics of it, and because it is a simpler process to use the time dilatation effect for this purpose, that is the approach adopted.

Prior to this however, it is necessary to inject a note of caution. The development that is to be presented here, involves a new idea concerning a direct relationship between space, time and matter. This relationship is one for which there does not exist, at the moment, any supporting observational nor experimental evidence. Nor does a means of conducting suitable observational or experimental research to verify the hypothesis seem clear. Consequently, it must be regarded as speculative, and may be the subject of considerable refinement.

In the interests of brevity, unless necessary for complete clarity, a parameter will only be defined in this paper, if it has not previously been so in either [1], [2] or [4], with which familiarity is assumed.

2 The Nature of Time in a Relativistic Domain.

2.1 Definitions.

As well as a preamble to the further developments presented herein, by providing a more in depth treatment on how time is generated in a Relativistic Domain, this section also augments certain definitions concerning time, given in [1] and [2].

Note that the symbol Δ has been introduced in this paper to represent a parametric elemental. This is to enable a distinction to be made between an element of time, and the symbology for temporal rates.

In any Relativistic Domain, time is generated as a result of motion along the temporal axis. In the Domain D_0, Pseudo-Euclidean Space-Time, with a temporal velocity of c, Fig 2.1 shows that motion on the temporal axis over an elemental distance Δx_0, creates an element of time Δt in the spatial part of the Domain.

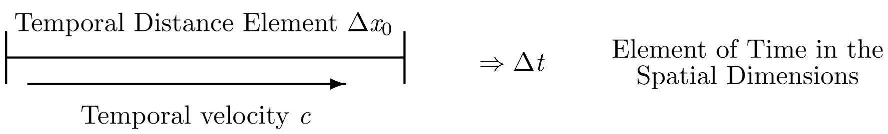

Fig. 2.1 - Generation of Time in Pseudo-Euclidean Space-Time D_0.

So that in $\mathbf{D_0}$.

$$\Delta t = \frac{\Delta x_0}{c} \tag{2.1}$$

In the Domain $\mathbf{D_1}$, the gravitational space-time, with a temporal velocity of cu, Fig. 2.2 shows that motion on the temporal axis over an elemental distance Δx_0, creates an element of time $\Delta\tau$ in the spatial part of the Domain.

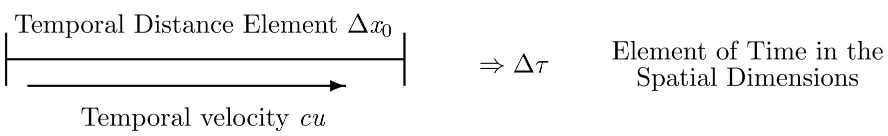

Fig. 2.2 - Generation of Time in Gravitational Space-Time, $\mathbf{D_1}$.

So that in $\mathbf{D_1}$.

$$\Delta\tau = \frac{\Delta x_0}{cu} \tag{2.2}$$

and therefore from (2.1) and (2.2)

$$\frac{\Delta\tau}{\Delta t} = \frac{1}{u} \tag{2.3}$$

Eq.(2.3) is the ratio of an element of time in $\mathbf{D_1}$ to that in $\mathbf{D_0}$. Because *u<1*, this shows that an element of time in $\mathbf{D_1}$ is longer than an element of time in $\mathbf{D_0}$. However, because both elements have the same value, time in $\mathbf{D_1}$ passes more slowly than it does in $\mathbf{D_0}$. Consequently, from (2.3)

$$1\ sec\ in\ \mathbf{D_0} \quad \equiv \quad u\ sec\ in\ \mathbf{D_1} \tag{2.4}$$

Determination of the rate of passage of time in $\mathbf{D_1}$ compared to $\mathbf{D_0}$ is accomplished via the following simple exercise.
In moving from one specifically defined point to another on the temporal axis, a body in the spatial part of $\mathbf{D_1}$ will experience a passage of time of

$$d\tau = u\ sec \tag{2.5}$$

During the above event the amount of time that passes in $\mathbf{D_0}$ is, by virtue of (2.4)

$$dt = 1\ sec \tag{2.6}$$

Therefore the rate of passage of time in $\mathbf{D_1}$, compared to $\mathbf{D_0}$ is from (2.5) and (2.6)

$$\frac{d\tau}{dt} = u \tag{2.7}$$

This was also derived in [1] via a slightly different process where u was thereby defined as the Temporal Rate of $\mathbf{D_1}$.

Finally, in the gravitational space-time $\mathbf{D_1}$, it is seen that because the passage of time is a function of the parameter u, which has, in [1], been shown to be a dimensionless function of the spatial radial distance σ. The passage of time in $\mathbf{D_1}$ is therefore also a function of this spatial distance.

2.2 Temporal Flow.

In previous papers [1] and [2], temporal velocity was always referred to as the velocity of a spatial point, or material object, on the temporal axis. However, because temporal velocities in $\mathbf{D_1}$ are a function of the spatial variable σ, if the above were the case, it would mean that different points in the spatial part of the Domain would exist at different locations on the temporal axis. Furthermore, as the temporal motion continued, different spatial points would move further apart on this axis. This is equivalent to 'travelling in time' which is obviously not within observed experience. Consequently, the above view of the nature of temporal velocity needs to be more precisely defined such that it satisfies both observed events and the requirements of gravitational theory. Consider Fig. 2.3.

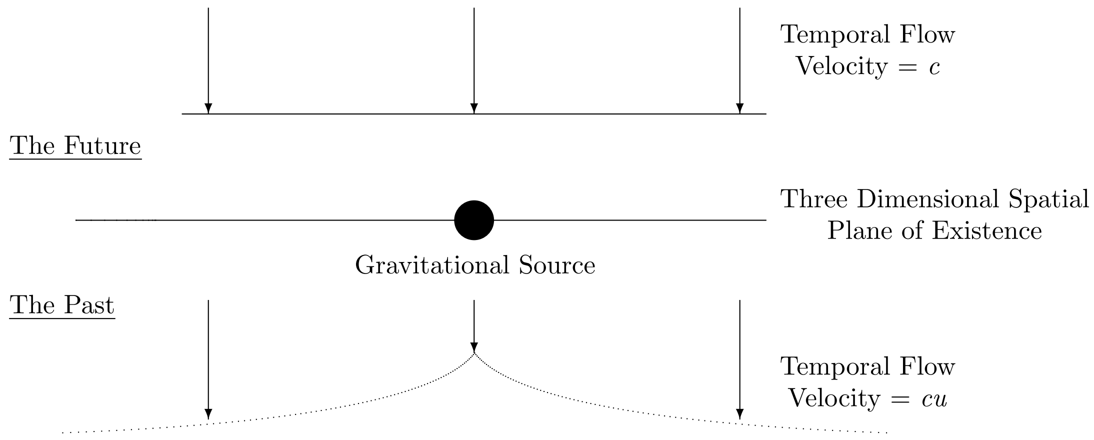

Fig. 2.3 - The Nature of Temporal Flow

It is proposed that temporal flow is a motion of the temporal dimension from the 'future' to the 'past', relative to a three dimensional spatial plane of existence on which all material bodies co-exist. In the future part of the dimension, the velocity of temporal flow is constant throughout at the value c. As the flow passes the spatial plane of existence, if a material body is present, the flow is altered in the manner described later in this paper such that the velocity is reduced. Thus, the velocity of the flow in the past region of the temporal dimension is no longer constant but becomes, in accordance with the parameter u, a function of the spatial distance from the source in all radial directions.

In this way all material bodies and spatial points in the spatial part of the Domain, can remain on the same three dimensional spatial plane of existence, but still experience a spatial variability in the rate of the passage of time, i.e. time dilatation.

It is obvious that, in the absence of material bodies on the plane of existence, i.e. in $\mathbf{D_0}$, temporal flow is at the constant velocity c throughout the entire Domain both future and past. It should also be remembered that should a material body gain a spatial velocity, then by virtue of the criterion of existence in the Domain, this will also result in a reduction in the rate of passage of time experienced by that body. This however, only affects the material body itself and not its internal or surrounding space.

The proposed nature of temporal flow presented above represents a more acceptable view in satisfying both the mathematical requirements of gravitation and observable facts. However, because the further mathematical analysis to be pursued is not compromised, and in the interests of consistency with previous papers, temporal velocity will continue in this paper to be referred to as that of material bodies and/or spatial points in the temporal direction.

The ensuing sections of this paper present a logical and mathematical description of a mechanism by which time dilatation, and subsequently the gravitational

Acceleration Potential, is generated by a gravitational source.

3 Time Dilatation.

3.1 Basic Concepts.

It has been shown in Section 2 that the passage of time experienced by all material bodies within the three spatial dimensions of $\mathbf{D_1}$, is due to the manner in which their velocity in the temporal direction is manifested in those dimensions, viz. from (2.2)

$$\Delta\tau = \frac{\Delta x_0}{cu} \tag{3.1}$$

It was also shown in Section 2 that the temporal rate of $\mathbf{D_1}$ with respect to $\mathbf{D_0}$ is, from (2.7)

$$\frac{d\tau}{dt} = u \tag{3.2}$$

Because time in $\mathbf{D_1}$, is by virtue of (3.1), derived from the temporal velocity of all spatial points within it, it is the actual or effective variation of this velocity, via the non-unity value of u, that results in the variation of the passage of time in the spatial dimensions. Clearly therefore, the time dilatation effect in $\mathbf{D_1}$ is expressed by its temporal rate u.
For all spatial points external to the gravitational source, the value of u was shown, in [1] Eq.(4.7), to be

$$u = \left(1 - \frac{2\alpha}{\sigma}\right)^{1/2} \tag{3.3}$$

where, as in [1],

> α is the gravitational radius of the source.
>
> σ is the radial distance from the centre of the source, to a point external to it.

Eq.(3.3) shows that the temporal rate in $\mathbf{D_1}$ external to the source, is a non-dimensional inverse function of the radial distance σ from its centre. Now, the generation of u can only take place within the physical constraints of the gravitational source, and it is therefore necessary that the mechanism responsible be shown to describe the equivalent of (3.3) within the source, as well as (3.3) itself outside the source. The equivalent of (3.3) within the source would be expected to exhibit the same form as (3.3) for reason of continuity at the boundary, the surface of the source.

To attempt an immediate discussion or mathematical development of this mechanism from first principles would require a number of somewhat speculative and complex assumptions concerning space, time and matter, assumptions which, at such an early stage in the procedure, would be impossible to adequately support. Fortunately however, it is possible to circumvent such an approach because the form for gravitational acceleration internal to a source, can be derived by using only one simple assumption. The result so obtained can then be used to derive a mathematical model for time dilatation in a more rigorous manner. The derivation of gravitational acceleration internal to a source is the subject of the next section.

3.2 Gravitational Acceleration Within a Source.

To effect this short derivation, it is assumed that gravitational acceleration, both inside as well as outside a source, is of the form

$$a = -\frac{\gamma(mass)}{(distance)^2} \tag{3.4}$$

With this assumption, it has been shown in [3], pp 9/10, that gravitational acceleration both internal and external to a thin spherical shell is given by

Internal

$$a_{int} = 0 \tag{3.5}$$

External

$$a_{ext} = -\frac{\gamma m_{sh}}{l^2} \tag{3.6}$$

where

m_{sh} is the total mass of the shell.

l is the distance of the measuring point from its centre.

A solid spherical mass may be considered as a series of concentric spherical shells, and therefore, the gravitational acceleration inside such a mass at a distance σ_i from its centre would be, from (3.5) and (3.6), simply

$$a_i = -\frac{\gamma m_i}{\sigma_i^2} = -\frac{\gamma m_g \sigma_i}{\sigma_g^3} \tag{3.7}$$

where

m_i is the mass of that part of the source inside σ_i.

m_g is the total mass of the gravitational source.

σ_g is the physical radius of the source.

Here, and for the remainder of this paper, the subscript i denotes a parametric value inside the source at $\sigma_{i.}$
The result (3.7) has also been derived in the reference.
Eq.(3.7) will be used later in developing a mathematical description of the mechanism causing time dilatation internal to the source. Prior to this however, the only three possible causes of this effect are reviewed for their likely applicability.

3.3 The Generation of Time Dilatation by a Gravitational Source.

3.3.1 Discussions.

Consider again (3.1). The variability of this relationship can only come about in one of three possible ways.

(i) Temporal velocity, cu, within the gravitational source is directly reduced by some interaction between the source, and the space-time in which it exists.

(ii) Temporal distance, Δx_0, is, via some interaction with the source, stretched such that an effective reduction in temporal velocity is simulated.

(iii) The source, via some interaction with the space-time in which it exists, generates a small spatial velocity along all radius vectors from the centre of the source. This spatial velocity, via the criterion of existence within $\mathbf{D_1}$, then results in a corresponding reduction in temporal velocity.

The lack of measurable interaction between the spatial and temporal dimensions of $\mathbf{D_1}$ prevents the proof or disproof of any of these potential causes, it being only possible, via mechanical means, to measure the passage of time in units related to astronomical or atomic events. However, each of the above potential causes can be logically examined to enable an assessment to be made of which is the more likely.

(i) Discussion of the Direct Reduction of Temporal Velocity.

Firstly, in the assessment of (i) above, for temporal velocity within the source to be the subject of a direct reduction, the interaction between the source and the fabric of the space time continuum, must be of the form of a resistance to motion in the temporal direction. The precedent for this is that it has been shown in both [1] and [4] that forces can exist in the temporal direction. The difference here is that the supposed forces causing the reduction in temporal velocity, would exist naturally in the temporal direction and not be due to, or associated with, any action or event in the spatial part of the Domain. This leads to a number of difficulties. The mathematical development for such a mechanism must lead to an expression similar to (3.3) for u inside the source. In the re-defined temporal flow in Section 2, the flow in the future part of the Domain would represent the unity term in such an expression. Other terms constituting the reduction from unity of u inside the source, would then be the result of the resistance to temporal motion inherent in the material body of the source. However, it is stated here without proof, that a hypothetical temporal forcing function, applied to any mass in any Relativistic Domain, including that resulting from a resistance to temporal motion, would not result in a change in temporal velocity, but instead a change in mass. This has already been effectively demonstrated in [1] and [4], but will also be discussed in more detail in a future paper. This is the first difficulty. Irrespective of this however, a further problem exists concerning space outside of the source where time dilatation is also in effect. Outside of the source there is nothing to interact with a temporal resistive force to create a reduction of temporal velocity. How the effect is promulgated outside of the source in this scenario is therefore unclear.

From the above discussion, this concept for the generation of time dilatation in $\mathbf{D_1}$ would therefore be a speculative, complex mechanism exhibiting a number of fundamental difficulties, which could not be adequately nor completely explained. It is therefore considered to be an unlikely mechanism for the generation of this effect.

(ii) Discussion of the Extension of Temporal Distance.

In the assessment of (ii) above it is noted that there appears to be a precedent in [1] Eq.(4.18), where, compared with the Domain $\mathbf{D_0}$, it is shown that an apparent small extension of spatial distance exists along all radius vectors from the centre of a gravitational source. This effect is discussed further in Appendix A to this paper. If the same effect were to occur on the temporal axis, due to the presence of the source, then (3.1) should really be written

$$\Delta\tau = \frac{\Delta x_0/u}{c} \tag{3.8}$$

and the term u would be associated with the incremental distance Δx_0, and not the velocity constant c. Consequently, temporal velocity would remain at the constant value c throughout the temporal dimension as it is in $\mathbf{D_0}$. Once again, there are difficulties with this approach. Firstly, the results of all previous analyses in [1] and [4] have been obtained by associating the parameter u with the velocity constant c, thus inferring that there is a real reduction in temporal velocity. The results of these previous analyses could not be fully realised if (3.8) was the true relationship for the passage of time in $\mathbf{D_1}$. Also, because the time dilatation effect appears outside the source, the stretching of temporal distance would also have to occur in this region. It is similarly unclear as to how this would be effected.
Consequently, it is also considered unlikely that this mechanism is the origin of time dilatation.

(iii) Discussion of the Generation of a Small Spatial Velocity.

Elimination of the above two possibilities leaves this concept as the potential cause of time dilatation in $\mathbf{D_1}$. Consequently, it is necessary that it be capable of adequate logical description, possessing none of the problems inherent in the previous two ideas. In particular, this concept must contain a clear mechanism to promulgate both the time dilatation effect, and the Acceleration Potential, outside the source.
The only precedent for this hypothesis is the mathematical form for u external to the source. This is shown as (3.3). Assume now that this relationship can also be expressed in the same form as [2] Eq(2.5), viz.

$$u = \left(1 - \frac{v_\sigma^2}{c^2}\right)^{1/2} \tag{3.9}$$

where v_σ is a small spatial velocity generated by the gravitational source. It is proposed that for reasons of continuity a similar relationship would also hold inside the source. If v_σ was a real velocity, and was to lead to the generation of time dilatation and the Acceleration Potential of $\mathbf{D_1}$, it would, because of the nature of gravitation, have to exist along all radius vectors from the centre of the source. Therefore, it could not be a velocity that was associated with the material body of the gravitational source itself, and the only possible alternative is that v_σ would therefore have to be a velocity of the very fabric of space, both within and surrounding the gravitational source. This is a new concept concerning the inter-relationship of space, time and matter, for which no supporting observational or experimental evidence exists. Nevertheless, it is proposed that v_σ is indeed a real velocity as described above, and which will be shown herein to be capable of causing both time dilatation and the Acceleration Potential of $\mathbf{D_1}$. It is further proposed that the generation of this "velocity of space", is effected by an expansion of space from within the source via a transition from the temporal dimension to the spatial, as the source motion along the temporal axis proceeds. The expansion takes place as an infinitely close series of spherical spatial wavefronts emanating from the centre of the source, and v_σ is the wavefront linear expansion velocity along all radius vectors outside the source. The equivalent inside the source is designated $v_{i\sigma}$. Both v_σ and $v_{i\sigma}$ by virtue of the criterion of existence in $\mathbf{D_1}$, result in a corresponding reduction in source temporal velocity thereby causing the time dilatation effect. Furthermore, in order for this spatial expansion velocity to be capable of producing the Acceleration Potential of $\mathbf{D_1}$, it is proposed, and will be shown, that v_σ and $v_{i\sigma}$ both exhibit a radial spatial gradient. Also, because gravitational acceleration is a function of the mass of the source, so therefore must v_σ and $v_{i\sigma}$ be, together with the spatial expansion and time dilatation effects they produce. One important consequence of this concept is that the resulting expansion of space, Acceleration Potential and time dilatation effect, both internal and external to the source, will be shown to occur with no discontinuity at the boundary.

To complete the discussion of this proposal, it is necessary to consider whether there is an energy transference or dissipation occurring during the process. As far as is currently known, space itself is not constructed of any form of distributed energy field, and the only energy that exists within it is particulate matter energy, and electromagnetic energy that is transmitted through it. It is not known to react directly with any form of energy in the space-time continuum currently envisaged. This suggests that the expansion of space by a gravitational source is not fuelled by any form of energy conversion from the source itself. Also supporting this view is that it has not been possible to construct a mathematical derivation, in which the time dilatation effect is produced by an energy conversion process within the source. It is therefore considered that this process is one in which the energy contained within the source is not involved, and the process, while proportional to the source mass, is governed only by its temporal motion.

While the above discussion, accepting the proposals made therein, provide a logical dissertation on a mechanism for the generation of time dilatation and the Acceleration Potential of $\mathbf{D_1}$, to ensure a degree of scientific credibility, it is necessary to provide proper mathematical support. Such support must enable a rigorous derivation of v_σ and $v_{i\sigma}$ from a basic consideration of the fundamental parameters involved. This is the subject of the next section.

3.3.2 The Mathematical Derivation of Time Dilatation by a Gravitational Source.

Prior to its derivation proper, it is first noted that the magnitude of the linear expansion velocity, after it has promulgated to the outside of the source, can easily be obtained by equating (3.3) and (3.9) and solving for v_σ thus

$$v_\sigma = \left(\frac{2\gamma m_g}{\sigma}\right)^{1/2} \tag{3.10}$$

It will also be obvious that the differential of (3.10) with respect to time in $\mathbf{D_1}$, equates to the Acceleration Potential of that Domain outside the source. This will be confirmed in Section 3.3.3.

While (3.10) gives the correct form for v_σ, the derivation just summarised does not provide any insight into the manner in which it is generated. To accomplish this, it is necessary to start from a more fundamental position.

Thus to start this development proper, it is necessary to determine the criterion that controls the whole process. To do this it is first noted that the parameter γ has units of

$$\frac{(length)^4}{(Force)\,(time)^4}$$

or

$$\frac{(length)^3}{(time)^2\,(Mass)}$$

Thus any term of the form γ*(mass)*, will have units *(length)*^{3}*/(time)*^{2}, i.e. a second order rate of change of volume. Therefore, because this product, γ*(mass)*, is predomi-

nant throughout gravitational theory, it is proposed that the generation of the spatial expansion within the source, can be described by some function of its second order rate of change of generated volume.
At this point, to ease the development process, the situation outside of the source will be considered first.

(i) Generation of Time Dilatation Outside the Gravitational Source.

Firstly, because it has been established earlier that the source does not dissipate energy in this process, any variation in the mass of the source for other reasons will be independent of it, and therefore a mathematical development to describe it as it appears outside the source, can treat the source mass as effectively constant. For this reason therefore, the second order rate of change of generated volume outside the source will also be constant, and this fact may be used to set up the following describing equation

$$\frac{d^2W}{d\tau^2} = K_0\gamma m_g \tag{3.11}$$

where

> W is the spatial volume generated by the gravitational source external to itself.
>
> K_0 is a dimensionless constant of proportionality.

And the solution of (3.11) must result in (3.10) as the linear velocity of expansion.
First, expand the second order differential in terms of its first and second order radial linear derivatives. e.g. with

$$W = \frac{4}{3}\pi\left(\sigma^3 - \sigma_g^3\right) \tag{3.12}$$

this differentiates to

$$\frac{d^2W}{d\tau^2} = 4\pi\sigma^2\frac{d^2\sigma}{d\tau^2} + 8\pi\sigma\left(\frac{d\sigma}{d\tau}\right)^2 \tag{3.13}$$

so that substitution into (3.11) gives

$$4\pi\sigma^2\frac{d^2\sigma}{d\tau^2} + 8\pi\sigma\left(\frac{d\sigma}{d\tau}\right)^2 = K_0\gamma m_g \tag{3.14}$$

To solve this for the first order linear derivative, first insert, from [1] Eq(4.2) for the second order linear derivative, (see also Section 3.3.3).

$$\frac{d^2\sigma}{d\tau^2} = -c^2u\frac{du}{d\sigma} = -\frac{\gamma m_g}{\sigma^2} \tag{3.15}$$

and solve for the first order derivative of σ thus

$$\left(\frac{d\sigma}{d\tau}\right)^2 = \frac{K_0\gamma m_g}{8\pi\sigma} + \frac{\gamma m_g}{2\sigma} \tag{3.16}$$

Now differentiate (3.16) with respect to time to give

$$\frac{d^2\sigma}{d\tau^2} = -\frac{K_0\gamma m_g}{16\pi\sigma^2} - \frac{\gamma m_g}{4\sigma^2} \tag{3.17}$$

Again inserting (3.15) and solving for K_0 gives

$$K_0 = 12\pi \tag{3.18}$$

Substituting this back into (3.16) then finally gives

$$v_\sigma = \frac{d\sigma}{d\tau} = \left(\frac{2\gamma m_g}{\sigma}\right)^{1/2} \tag{3.19}$$

The desired result. This is the linear spatial expansion rate emanating from the gravitational source and which exhibits a spatial variation such that its first derivative with respect to time results in (3.15), the Acceleration Potential of $\mathbf{D_1}$.
This small spatial velocity, by virtue of the criterion of existence in $\mathbf{D_1}$, causes a corresponding reduction of the temporal velocity of all spatial points outside of the source thus, via effectively [2] Eq(2.5)

$$\frac{dx_0}{d\tau} = \left(c^2 - v_\sigma^2\right)^{1/2} = c\left(1 - \frac{2\gamma m_g}{\sigma c^2}\right)^{1/2} = c\left(1 - \frac{2\alpha}{\sigma}\right)^{1/2} = cu \tag{3.20}$$

and therefore the time dilatation effect as shown by the final term of (3.20) in conjunction with (3.1).
The situation inside the source will now be considered.

(ii) Generation of Time Dilatation Inside the Gravitational Source.

Eq(3.14) may also be used as the describing equation inside the source except that now, the right hand side is no longer a constant. The right hand side is therefore unknown in this situation and consequently (3.14) must be modified to accommodate this. Therefore initially write

$$4\pi\sigma_i^2\frac{d^2\sigma_i}{d\tau^2} + 8\pi\sigma_i\left(\frac{d\sigma_i}{d\tau}\right)^2 = f() \tag{3.21}$$

where $f()$ is some function involving γ, σ_i, the position of measurement, and m_g the mass of the source. A solution for the first linear derivative can still be found by using the method adopted above. First insert (3.7) for the second order derivative and solve for the first order to give

$$\left(\frac{d\sigma_i}{d\tau}\right)^2 = \frac{f()}{8\pi\sigma_i} + \frac{\gamma m_g\sigma_i^2}{2\sigma_g^3} \tag{3.22}$$

Differentiating with respect to time then gives

$$2\frac{d^2\sigma_i}{d\tau^2} = \frac{df()/d\sigma_i}{8\pi\sigma_i} - \frac{f()}{8\pi\sigma_i^2} + \frac{\gamma m_g\sigma_i}{\sigma_g^3} \tag{3.23}$$

Inserting (3.7) again for the second order derivative permits a simple differential equation in $f()$ to be set up thus

$$\frac{df()}{d\sigma_i} - \frac{f()}{\sigma_i} = -\frac{24\pi\gamma m_g\sigma_i^2}{\sigma_g^3} \tag{3.24}$$

this equation is linear and can be solved with an integrating factor of $1/\sigma_{i.}$ The result is

$$f() = -\frac{12\pi\gamma m_g\sigma_i^3}{\sigma_g^3} + k\sigma_i \tag{3.25}$$

k is the constant of integration and can be determined by inserting (3.25) back into (3.21) and repeating the above process as follows, (or inserting (3.25) directly into (3.22)).

$$4\pi\sigma_i^2 \frac{d^2\sigma_i}{d\tau^2} + 8\pi\sigma_i \left(\frac{d\sigma_i}{d\tau}\right)^2 = -12\pi\gamma m_g \frac{\sigma_i^3}{\sigma_g^3} + k\sigma_i \tag{3.26}$$

Inserting (3.7) and solving for the first order derivative gives

$$\left(\frac{d\sigma_i}{d\tau}\right)^2 = -\frac{\gamma m_g \sigma_i^2}{\sigma_g^3} + \frac{k}{8\pi} \tag{3.27}$$

From the previous analysis for the situation outside the source, it is known that at the lower boundary, the surface of the source, when $\sigma = \sigma_g$

$$\left(\frac{d\sigma}{d\tau}\right)^2_{\sigma=\sigma_g} = \frac{2\gamma m_g}{\sigma_g} \tag{3.28}$$

and, for reason of continuity, this also applies at the upper boundary for σ_i. Therefore in (3.27) putting $\sigma_i = \sigma_g$ and inserting (3.28) gives the value for k as

$$k = \frac{24\pi\gamma m_g}{\sigma_g} \tag{3.29}$$

So that in (3.27) this finally gives

$$v_{i\sigma} = \frac{d\sigma_i}{d\tau} = \left[\frac{3\gamma m_g}{\sigma_g} - \frac{\gamma m_g \sigma_i^2}{\sigma_g^3}\right]^{1/2} \tag{3.30}$$

for the linear rate of spatial expansion inside a gravitational source. The derivative of (3.30) with respect to time then gives (3.7) for the gravitational acceleration within the source.
The relationship for the temporal rate and therefore time dilatation inside the source, is then obtained in the same manner as in the previous section, viz.

$$\begin{aligned} \frac{dx_0}{d\tau} &= \left(c^2 - v_{i\sigma}^2\right)^{1/2} \\ &= c\left(1 - \frac{3\gamma m_g}{c^2\sigma_g} + \frac{\gamma m_g \sigma_i^2}{c^2\sigma_g^3}\right)^{1/2} \\ &= c\left(1 - \frac{3\alpha}{\sigma_g} + \frac{\alpha\sigma_i^2}{\sigma_g^3}\right)^{1/2} \\ &= cu_i \end{aligned} \tag{3.31}$$

3.3.3 The Acceleration Potential of $\mathbf{D_1}$*.*

In [1] the Acceleration Potential of $\mathbf{D_1}$ was primarily quoted in terms of its temporal rate and therefore, that form will also be derived here. For the situation outside the source, from (3.20)

$$u = \left(1 - \frac{2\gamma m_g}{c^2\sigma}\right)^{1/2} \tag{3.32}$$

Differentiating (3.32) with respect to σ gives

$$\frac{du}{d\sigma} = \frac{\gamma m_g}{c^2\sigma^2}\left(1 - \frac{2\gamma m_g}{c^2\sigma}\right)^{-1/2} \tag{3.33}$$

so that

$$-c^2 u\frac{du}{d\sigma} = -\frac{\gamma m_g}{\sigma^2} \tag{3.34}$$

The negative signs here have been inserted for directional correctness. Clearly (3.34) is identical with (3.15) and the analytical results of [1] Section 4.
A similar process applied inside the gravitational source from (3.31) yields for the Acceleration Potential, (3.7) thus,

$$-c^2 u_i\frac{du_i}{d\sigma_i} = -\frac{\gamma m_g \sigma_i}{\sigma_g^3} \tag{3.35}$$

It is important to note that the Acceleration Potential of $\mathbf{D_1}$, both inside and outside the source is not in the true sense a time dependent variable. It exists as a spatial field due to the spatial variabilities of both $v_{i\sigma}$ and v_σ.
Note that in (3.31) when $\sigma_i = 0$, the value of u_i at the very centre of the source is given by

$$u_i\big|_{\sigma_i=0} = \left(1 - \frac{3\alpha}{\sigma_g}\right)^{1/2} \tag{3.36}$$

which has implications regarding a limiting value for σ_g. This together with other 'close proximity' effects will form the subject of a future paper.

3.3.4 Derivation of the Field Propagation Equations.

The derivation of the previous Sections have provided a mathematical model for the manner in which the Acceleration Potential and the time dilatation effect are generated by a gravitational source. From this it is quite clear how these effects are promulgated to the space surrounding the source. To augment that understanding, this Section will provide the field equations describing that propagation. It is to be noted that because the space-time continuum within which this new gravitational theory has been developed, $\mathbf{D_1}$, is a linear space-time continuum, the field equations derived here will be identical to those of Newtonian theory.
The process of derivation is started from (3.9) as follows, (thus the situation outside the physical boundaries of the source are to be considered first)

$$u = \left(1 - \frac{v_\sigma^2}{c^2}\right)^{1/2} \tag{3.37}$$

Solving for $v_\sigma^2/2$

$$\frac{v_\sigma^2}{2} = \frac{c^2}{2}\left(1 - u^2\right) \tag{3.38}$$

Call this term U_σ, which can be identified as similar to the Newtonian Potential of classical theory, then

$$\frac{\partial U_\sigma}{\partial\sigma} = -c^2 u\frac{\partial u}{\partial\sigma} \tag{3.39}$$

From (3.34), this can be expressed as

$$\frac{\partial U_\sigma}{\partial\sigma} = -\frac{\gamma m_g}{\sigma^2} \tag{3.40}$$

and thus

$$\frac{1}{\sigma^2}\frac{\partial}{\partial\sigma}\left(\sigma^2\frac{\partial U_\sigma}{\partial\sigma}\right) = 0 \tag{3.41}$$

The L.H.S. of (3.41) is the Laplace equation for a spherically symmetrical mass. Therefore, the field propagation equation outside of the source may be stated as

$$\nabla^2 U_\sigma = 0 \tag{3.42}$$

Also, because this means that the Acceleration Potential can be stated as

$$\mathbf{A}_\sigma = \frac{\partial U_\sigma}{\partial\sigma}\sigma = \nabla U_\sigma \tag{3.43}$$

then by vector theory

$$\nabla\mathbf{x}\mathbf{A}_\sigma = 0 \tag{3.44}$$

and the Acceleration Potential vector field outside of the gravitational mass is irrotational as is implicit in the definition of the gravitational space-time continuum. Internal to the source, the equivalent of (3.37) is

$$u_i = \left(1 - \frac{v_{i\sigma}^2}{c^2}\right)^{1/2} \tag{3.45}$$

and therefore as in the process above outside the source

$$\frac{v_{i\sigma}^2}{2} = \frac{c^2}{2}\left(1 - u_i^2\right) \tag{3.46}$$

calling this $U_{i\sigma}$, the internal equivalent of U_σ then as above

$$\frac{\partial U_{i\sigma}}{\partial\sigma_i} = -c^2 u_i\frac{\partial u_i}{\partial\sigma_i} \tag{3.47}$$

so that from (3.7)

$$\frac{\partial U_{i\sigma}}{\partial\sigma_i} = -\frac{\gamma m_g\sigma_i}{\sigma_g^3} \tag{3.48}$$

and again following the derivation outside the source

$$\frac{1}{\sigma_i^2}\frac{\partial}{\partial\sigma_i}\left(\sigma_i^2\frac{\partial U_{i\sigma}}{\partial\sigma_i}\right) = -4\pi\gamma\rho_g \tag{3.49}$$

Here the L.H.S. is Poisson's equation for a spherically symmetrical mass, and therefore, the field propagation equation inside of the source may be stated as

$$\nabla^2 U_{i\sigma} = -4\pi\gamma\rho_g \tag{3.50}$$

In (3.49) and (3.50) ρ_g is the average density of the gravitational source. Therefore, again by vector theory

$$\mathbf{A}_{\mathbf{i}\sigma} = \frac{\partial U_{i\sigma}}{\partial\sigma_i}\sigma_{\mathbf{i}} = \nabla U_{i\sigma} \tag{3.51}$$

and so once again

$$\nabla\mathbf{x}\mathbf{A}_{\mathbf{i}\sigma} = 0 \tag{3.52}$$

and the Acceleration Potential vector field inside the gravitational source is also irrotational, again an expected result.

Above it was stated that the parameter U_σ was similar to the Newtonian Potential U of classical theory. Their exact relationship is as follows. From (3.40)

$$U_\sigma = \frac{\gamma m_g}{\sigma} \tag{3.53}$$

and with

$$U = \frac{\gamma m_g}{r} \tag{3.54}$$

then from (3.53) and (3.54)

$$U_\sigma = U\frac{r}{\sigma} \tag{3.55}$$

and therefore the only difference between the Newtonian and Relativistic Domain gravitational potentials is a relativistic one. Again in view of the definition of the gravitational space-time continuum used in this new theory, this is an expected result.

4 A Physical Interpretation of the Results.

The purpose of this section is to augment the discussions in Section 2, and the mathematical derivations in Section 3, with a further interpretation of the results in a physical sense. This, it is hoped, will assist in the understanding of the process of generation of the gravitational effect presented here, within the Relativistic Domain $\mathbf{D_1}$. However, it is again emphasised that because of the lack of observational evidence, this interpretation must also be considered as somewhat speculative.

All points in space, including any material bodies that occupy them, exist within a Relativistic Domain according to one simple criterion. The vector sum of their velocities along all four Domain axes must always be equal to a specific value. In previous papers this was termed the Existence Velocity of the Domain. In the Domain $\mathbf{D_0}$, Pseudo-Euclidean Space-Time, the magnitude of Existence Velocity is the constant velocity parameter c. In general, the greater part of Existence Velocity will be along the temporal axis, and the passage of time in the spatial part of the Domain is then given by the ratio of the distance travelled along the temporal axis, to the velocity along that axis. When a material body is present in $\mathbf{D_0}$, it interacts with the Domain thereby changing it to produce the gravitational Domain $\mathbf{D_1}$. It is proposed that this occurs in the following manner.

As the motion of the gravitational source along the temporal axis proceeds, a transition takes place from the temporal dimension to the three spatial dimensions such that, an expansion of the three spatial dimensions takes place along all radius vectors from the centre of the source. The transition enters the spatial part of the Domain at the very centre of the source, and expands outwards with the linear expansion velocity $v_{i\sigma}$. The initial value of the linear expansion velocity at the centre of the source is given by the constant term in (3.30). As the expansion moves away from the centre in an infinitely close series of wavefronts, the linear expansion velocity is subjected to a measure of attenuation. Firstly, a degree of spatial re-absorption appears to take place within the source, causing a deceleration of $v_{i\sigma}$ as represented by the second, variable term in (3.30). This re-absorption, being a function of distance from the centre of the source, increases as the expansion proceeds.

Upon reaching the outer periphery of the source, the re-absorption stops and the linear expansion velocity at this point, and beyond, simplifies to that given by (3.19). The only attenuation after this point is that resulting from the inverse distance dependency. The expansion continues on in this manner into the space surrounding the source.

It should be noted that in the derivation of the spatial linear expansion velocities, v_σ and $v_{i\sigma}$ in (3.19) and (3.30), the positive roots have been taken. If the negative roots are taken, negative gravitational results would be obtained and therefore both positive and negative values of these parameters provide acceptable solutions. The main difference of course is that in the case of negative roots, the terms, v_σ and $v_{i\sigma}$ then represent contraction velocities because in this case, the source would be generating a contraction of space. The importance of this will become evident in a future paper on the cosmological application of Relativistic Domain Theory.

Returning to the solution detailed here, using the positive roots of (3.19) and (3.30) to produce normal gravitation, the two principle consequences, applicable both inside and outside the source, are due to the spatial variability of the linear expansion velocity. The first is that this variability is the direct cause of gravitational acceleration in the form of the Acceleration Potential of the Domain. The second consequence concerns the criterion of existence within the Domain. The linear expansion velocity of all points both inside and outside the source, causes a corresponding reduction in their temporal velocity resulting in the basic time dilatation effect. Also, because the linear expansion velocity is a function of radial distance, so then is the temporal velocities of all spatial points, and thus via (3.1), the rate of the passage of time is also a function of radial distance, thereby constituting the radial variability of the time dilatation effect.

Finally, it is necessary to comment on other effects this spatial expansion might have on other material bodies in the vicinity. The Acceleration Potential and time dilatation discussed above have an additional combined effect. The combination produces the net acceleration experienced by a gravitating body as shown in [1] Eq(3.18), the equation of gravitational planar orbital motion. The Acceleration Potential causes the primary negative acceleration towards the source, while the spatial variability of time dilatation causes a velocity dependent positive acceleration. In normal astronomical situations the first of these terms is predominant. This is the only additional effect that would be experienced by other material bodies close to a gravitational source. In particular, the expansion of space itself would not have any further effect. This is because the expansion does not involve the dissipation of energy, and therefore cannot impose a mechanical force on other material bodies. The spatial expansion would simply flow past and through such bodies, and consequently their states of motion would not be altered by this part of the process.

It is possible however, that some absorbtion may take place as the spatial expansion from one source flowed through another close by, just as it does in the interior of the generating body. The result would be that the combined gravitational effect behind both bodies in line, would be very slightly less than the sum of their individual gravitational potentials.

5 Concluding Remarks.

Of the possible mechanisms that could be responsible for time dilatation and the Acceleration Potential of $\mathbf{D_1}$, the only one for which a creditable mathematical process can be derived, is that in which the gravitational source creates a localised spatial expansion, via transition from the temporal dimension. In particular, it clearly promulgates satisfactorily outside the source with no discontinuity at the boundary, the physical surface of the source. This is a new concept for which no experimental nor observational support exists, and has therefore to be regarded as speculative. However, the manner in which the parameter γ fits into the concept provides good circumstantial support to the idea, and indeed provides a physical meaning for the parameter γ itself.

The velocity v_σ appears on all radius vectors from the source, and as described appears because of the expansion of space emanating from it. As has been discussed, the only effects this process has on other material bodies is gravitational acceleration, and time dilatation. However, under certain conditions, it is possible that the spatial expansion process, over long enough periods of time, may have significant cosmological consequences. These will be discussed at length in a future paper.

One other significant point is the similarity of the field equations of the new theory to those Newtonian theory. This is due to the linearity of the space-time continuum in the new theory and its only difference to that of Newtonian theory being a relativistic one.

In this paper the Acceleration Potential has been shown to be due to the spatial variability of the linear velocity of the spatial expansion wavefront generated by the gravitational source. Time dilatation is in turn caused by the variation in local temporal velocity, due to the presence of the spatial expansion and the criterion for existence in $\mathbf{D_1}$. However, because the spatial expansion wavefront is a spatially distributed variable, so is the time dilatation effect and this in turn affects the net gravitational acceleration experienced by a free body in $\mathbf{D_1}$. This effect is fully expressed in the equation for planar gravitational motion in $\mathbf{D_1}$, viz. [1], Eq(3.18).
Thus the proper and complete cause of gravitational motion in $\mathbf{D_1}$, can now be formally stated as follows:-

(i) The primary cause is a negative acceleration produced by the Acceleration Potential of the Domain, which is, in turn, the result of the spatial variability of the linear radial velocity of a spatial expansion generated within the source as a result of its motion in the temporal direction.

(ii) The secondary cause is a positive acceleration due to the motion of the gravitating body through the varying temporal rate, resulting from the spatially distributed time dilatation effect.

Appendix A

The Relationship Between the Spatial Axes of $\mathbf{D}_1$ and $\mathbf{D}_0$.

Here it is necessary to confirm that the spatial axes of $\mathbf{D_1}$ are those axes fixed with their zero at the centre of the gravitational source, and whose radial variable outside the source is represented in this series of papers by the parameter σ. It does not mean axes attached to localised space.
For the situation external to the source, the relationship between these axes has been established as [1] Eq(4.18). An alternative form for it can be developed very easily starting once again from the equivalence of (3.3) and (3.9). thus

$$\left(1 - \frac{2\alpha}{\sigma}\right)^{1/2} = \left(1 - \frac{v_\sigma^2}{c^2}\right)^{1/2} \tag{A.1}$$

from which α can be obtained in terms of v_σ as

$$\alpha = \frac{\sigma v_\sigma^2}{2c^2} \tag{A.2}$$

Inserting (A.2) into [1] Eq(4.18) then gives

$$\sigma = r + \frac{\sigma v_\sigma^2}{2c^2} \tag{A.3}$$

and σ is then determined in terms of r as

$$\sigma = r\left(1 - \frac{v_\sigma^2}{2c^2}\right)^{-1} \tag{A.4}$$

which shows that the relationship between lengths on the respective axes of $\mathbf{D_1}$ and $\mathbf{D_0}$ outside the source, to be a simple relativistic one.
The comparable relationship within the source cannot exhibit any form of discontinuity at the boundary, i.e. the surface of the source. Therefore, this relationship must hold both at this boundary, and within the source. Thus with u_i from (3.31) re-written as

$$u_i = \left(1 - \frac{3\alpha}{\sigma_g} + \frac{\alpha_i}{\sigma_i}\right)^{1/2} \tag{A.5}$$

where

$$\alpha_i = \frac{\gamma m_i}{c^2} \tag{A.6}$$

and if (3.9), with $v_{i\sigma}$ substituted for v_σ, is equated to (A.5) and the above process applied, the result is

$$\sigma_i = r_i\left(1 - \frac{3\alpha}{\sigma_g} + \frac{v_{i\sigma}^2}{c^2}\right)^{-1} \tag{A.7}$$

This importantly shows that $\sigma_i = 0$ when $r_i = 0$.

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