GRAVITATION -

THE INVERSE SQUARE LAW

ANOMALY AND ITS RESOLUTION

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ABSTRACT.

In this paper the inverse square law of gravitation is shown to contain an anomaly insofar as it theoretically permits the gravitational field of a source of any size to extend to infinity. A solution is developed for the Relativistic Domain theory of gravitation which is subsequently also shown to be an unlikely contributory explanation for the Pioneer Anomaly.

CONTENTS.

1.0 Introduction.

In gravitational theory, the inverse square law that relates the strength of the gravitational field at any point, to the distance of that point from the centre of the source, is the central result of the theory. It is so in Newton's non-relativistic theory, Einstein's General Theory of Relativity, and the Relativistic Domain theory of gravitation that is the subject of this series of papers. However, as it stands, this law contains a serious anomaly, in that it does not go to zero at any finite distance from the source. Consequently, the gravitational field of any object, irrespective of its size or mass, can theoretically extend beyond the boundary of the entire Universe out to infinity. Albeit the strength of the field at such extreme distances would be infinitesimally small, such a phenomenon is considered to be impractical.

It is proposed that removal of this anomaly would require a fundamental modification of gravitational theory. Newton's theory cannot be so modified because this theory is based upon the concept of "action at a distance", and consequently it would not be possible to derive the necessary amendment from any logical hypothesis. To modify Einstein's General Theory of Relativity would probably require an appropriate amendment to the curvature tensor. However, such an amendment would have to be fully justifiable in descriptive as well as mathematical terms, and be fully derivable from fundamental concepts in conjunction with the rest of the theory. Such a task is left to those who support Einstein's theory.

The remaining purpose of this paper is to provide a modification to the Relativistic Domain theory of gravitation to remove the anomaly from that theory.

To fully appreciate the content of this paper, it is essential that references [1], [2] and [3] are read first. Accordingly, a parameter will only be defined if it has not already been so in the above references, or for absolute clarity.

2.0 The Inverse Square Law Anomaly.

2.1 Description of the Anomaly

The inverse square law anomaly applies only to that part of the gravitational field external to the source. The law is mathematically expressed as.

$$A_\sigma = -\frac{\gamma M_g}{\sigma^2} \tag{2.1}$$

where

- A_σ is the Acceleration Potential of the Relativistic Domain theory of gravitation.
- γ is Newton's constant of proportionality.
- M_g is the mass of the gravitational source.
- σ is the distance to the centre of the source from the point of measurement.

It is clear from (2.1) that A_σ can only go to zero when $\sigma \to \infty$.
To remove the anomaly, it is necessary to return to the fundamental cause of gravitation in this theory as developed in [1], [2] and [3].

2.2 Resolution of the Anomaly in the Relativistic Domain Theory of Gravitation.

The primary relationship central to the development of gravitation in this theory, as stated in [3], Eq.(3.11), and repeated here for convenience is, for the situation external to the source

$$\frac{d^2W}{d\tau^2} = K_0\gamma M_g \tag{2.2}$$

where K_0 was subsequently determined to be 12π. In (2.2) it is clear that the second order rate of change of the volume of the spherical spatial expansion wave is a constant, and so the wave can go on expanding ad infinitum. The inverse square law anomaly is the consequence of this.

To remove the anomaly, it is necessary to introduce into (2.2) a limiting restriction based upon a new characteristic of three dimensional space. It is consequently proposed that space possesses an attribute, such that a restriction proportional to the surface area of the spherical spatial expansion wave is generated. Such a concept is not without precedence in that it is similar to the spatial characteristic that the General Theory of Relativity must incorporate, in order to result in the curvature of space in the presence of ponderable matter.

Accordingly, the above expansion restriction would amend the second order rate of change of the volume of the expansion wave internal to the source thus,

$$\frac{d^2W_i}{d\tau^2} = 12\pi\gamma M_g\left(2\frac{\sigma_i}{\sigma_g} - \frac{\sigma_i^3}{\sigma_g^3} - \frac{4\pi k\sigma_i^2}{\sigma_g}\right) \tag{2.3}$$

where the final term in the brackets is the modification. The parameter k is a constant of proportionality with dimensions of cm^{-1}. The parameter σ_g appears in this term because, for an expansion wave of a given surface area, the degree of restriction would be inversely proportional to the size of the gravitational source. The other terms, (the unmodified equation), were obtained from [3], Eq.(3.26) with [3], Eq.(29) incorporated.

Eq.(2.3) can be written

$$\upsilon_{\sigma i}\frac{d}{d\sigma_i}\left(\sigma_i^2\upsilon_{\sigma i}\right) = 3\gamma M_g\left(2\frac{\sigma_i}{\sigma_g} - \frac{\sigma_i^3}{\sigma_g^3} - \frac{4\pi k\sigma_i^2}{\sigma_g}\right) \tag{2.4}$$

where

$\upsilon_{\sigma i} = \dfrac{d\sigma_i}{d\tau}$ is the radial velocity of the spatial expansion wave internal to the source.

Using the integrating factor σ_i^2 (2.4) becomes

$$\sigma_i^2\upsilon_{\sigma i}\frac{d}{d\sigma_i}\left(\sigma_i^2\upsilon_{\sigma i}\right) = 3\gamma M_g\left(2\frac{\sigma_i^3}{\sigma_g} - \frac{\sigma_i^5}{\sigma_g^3} - \frac{4\pi k\sigma_i^4}{\sigma_g}\right) \tag{2.5}$$

This now integrates immediately to give

$$\upsilon_{\sigma i} = \left(\frac{3\gamma M_g}{\sigma_g} - \frac{\gamma M_g \sigma_i^2}{\sigma_g^3} - \frac{24\pi\gamma M_g k \sigma_i}{5\sigma_g} \right)^{1/2} \tag{2.6}$$

as the modified radial velocity of the spatial expansion wave inside the source. As the wave reaches the surface of the source, (2.6) becomes

$$\upsilon_{\sigma g} = \left[\frac{2\gamma M_g}{\sigma_g} - \frac{24\pi k \sigma_g}{5} \left(\frac{\gamma M_g}{\sigma_g} \right) \right]^{1/2} \tag{2.7}$$

Eq.(2.7) has been written in this form because at this point, the source is no longer generating a spatial expansion wave, and so interpretation of σ_g in this equation varies. In the term $2\gamma M_g/\sigma_g$, σ_g is now merely the distance of the wave from the centre of the source and is therefore a variable which becomes σ as the wave progresses further. This also applies to the σ_g in the numerator of the second term. In the denominator of this second term however, σ_g is to be treated as a constant. This difference exists because of the origin of these terms. $2\gamma M_g/\sigma_g$ originates directly from the source, while the other term is due to the new attribute of space as previously described.
Hence as the wave progresses to the exterior of the source (2.7) becomes,

$$\upsilon_{\sigma} = \left[\frac{2\gamma M_g}{\sigma} - \frac{24\pi k \sigma}{5} \left(\frac{\gamma M_g}{\sigma_g} \right) \right]^{1/2} \tag{2.8}$$

The differential of (2.8) with respect to the time now yields the modified Acceleration Potential thus

$$A_\sigma = -\frac{\gamma M_g}{\sigma^2} - \frac{12\pi\gamma M_g k}{5\sigma_g} \tag{2.9}$$

Eq.(2.9) itself does not possess a real zero, but in (2.8), υ_σ goes to zero when

$$\sigma = \left(\frac{5\sigma_g}{12\pi k} \right)^{1/2} \tag{2.10}$$

Once again, as there is no generation of spatial expansion outside the source, υ_σ remains at zero for all values of σ greater than (2.10). Consequently, the Acceleration Potential is also zero at such values of σ.
Insertion of (2.10) into (2.9) the gives the minimum value of A_σ as

$$A_\sigma(\min) = -\frac{24\pi\gamma M_g k}{5\sigma_g} \tag{2.11}$$

It is at this value that the gravitational field abruptly ends and A_σ is zero thereafter.

It is necessary to ensure that the modification does not result in an anomaly inside the source. Therefore taking the time derivative of (2.6) gives the Acceleration Potential inside the source as

$$A_{\sigma i} = -\frac{\gamma M_g \sigma_i}{\sigma_g^3} - \frac{12\pi\gamma M_g k}{5\sigma_g} \tag{2.12}$$

and (2.12) clearly does not possess a zero for positive σ_i.
From (2.6) $\upsilon_{\sigma i}$ goes to zero when

$$\sigma_i^2 + \frac{24\pi k \sigma_g^2 \sigma_i}{5} - 3\sigma_g^2 = 0 \tag{2.13}$$

and the value of *k* will be shown to be such that (2.13), for a normal gravitational source, can be approximated so that

$$\sigma_i \approx \sqrt{3}\sigma_g \tag{2.14}$$

This zero is beyond the surface of the gravitational source and does not therefore represent an anomaly.

2.3 The Value of *k*.

The value of *k* cannot be determined theoretically, and so must be the subject of experimentation. The only "experimental" results available from which *k* could be determined are those represented by the so called Pioneer Anomaly. Both the Pioneer 10 and Pioneer 11 space probes were found to exhibit an unmodelled constant retardation of some 8.74E-8 cm/sec^2 as they left the Solar System in opposite directions. Full details of this anomaly is documented in [4]. The anomaly has been, and still is, the subject of many investigations, also documented in [4] and elsewhere. The cause currently favoured by investigators is a systemic one resulting from asymmetric thermal radiation, but this has yet to be proven. If instead, some or all of this retardation were the result of the modification to Relativistic Domain gravitation, then *k* could be determined as follows.
Taking the full value of the Pioneer Anomaly unmodelled retardation as stated above, (in the absence of further investigative results), and equating it to the modification constant term in (2.9) gives

$$\frac{12\pi\gamma M_g k}{5\sigma_g} = 8.74E-8\ cm/sec^2 \tag{2.15}$$

which re-arranges for *k* as

$$k = \frac{(3.64E-8)\sigma_g}{\pi\gamma M_g} \tag{2.16}$$

inserting the following values into (2.16)

$\gamma = 6.67E\text{-}8\ cm^4/g\ sec^4$
$M_g = 1.981E33\ g\ sec^2/cm$, (Mass of the Sun)
$\sigma_g = 6.96E10\ cm$, (Radius of the Sun)

gives

$$k = 6.11E-24\,cm^{-1} \tag{2.17}$$

as the value of *k*. However, using this value to determine the extent of the gravitation field of the Sun only produces a value of some 2,619A.U. A value much too small to account for the existence of the Oort Cloud, (≈ 100,000A.U.). Consequently, using the total of the Pioneer retardation anomaly to determine *k* is an exaggeration. Instead it is necessary to assume that the distance to the Oort Cloud in fact represents the extent of the of the Suns gravitaional filed and to then determine the appropriate value of *k* accordingly.
Thus equating (2.10) to this distance provides a value of *k*

$$k = 4.16E-27cm^{-1} \tag{2.18}$$

Inserting this value into (2.15) then gives the resulting accelerative retardation as

$$A_{ret} = 5.96E-11cm/\sec^2 \tag{2.19}$$

This is three orders of magnitude less than that suffered by the Pioneer spacecraft and is therefore not a significant contributory factor.

3.0 **Conclusions.**

It is clear that the modification to the Relativistic Domain theory of gravitation has resolved the inverse square law anomaly, inherent in the unmodified version and other gravitational theories.
The main point of interest, and possibly contention, is the nature of the modification in that it attributes to three dimensional space a physical characteristic similar to elasticity. As stated in the main text, such an attribute is not without precedence in that the General Theory of Relativity must also attribute a suitable physical characteristic to space, such that it can become curved by the presence of ponderable matter. Without this characteristic the General Theory would not exist as a proposed theoretical explanation for gravity. The characteristic attributed to space proposed here is not so critical, but nevertheless essential for the purpose intended. It cannot be directly associated with the gravitational source itself, because it would then have no external effect. Modification of the nature of space is the only way in which it can be effective external to the source. Other modifications have been tried, but none have given satisfactory results.

The specifics of the modification can be likened to the inflation of a rubber balloon in that resistance to inflation increases with the degree of inflation. Also, for a given diameter of inflation, a large balloon will present less resistance to further inflation than a small balloon. Hence, applied to three dimensional space, the modification restriction to spatial expansion is a function of the surface area of the spherical expansion wave divided by the radius of the source.

By using the distance to the Oort Cloud in the modification process, provides a value for the constant of proportionality which is considered a not unacceptable figure for such a spatial characteristic. Should objects more distant than the Oort Cloud be shown to be gravitationally bound to the Solar System, the value of k determined here will need to be amended. However, at most, such changes would be expected to have less than a single order of magnitude effect on any parameter derived here. Such parameters are discussed in Appendix A where the effects of the modification with respect to the gravitational fields of the Earth, the Sun and the entire Universe are presented.

APPENDIX A

Astronomical and Cosmological Consequences.

A.1 The Sun.

Insertion of (2.13) into (2.10) gives the extent of the Sun's gravitational field as

$$\sigma_S = 1.49E18\,cm \qquad \text{(A.1)}$$

in (2.10) a figure of 6.96E 10 for σ_g was used. Eq.(A.1), the distance to the Oort Cloud, 100,000A.U, shows that the Sun's gravitational field only extends to some 1% of the distance to the nearest star, Proxima Centuri.

A.2 The Earth.

The constant term in (2.9) for the Earth becomes

$$a_E = 1.78E-14\,cm/sec^2 \qquad \text{(A.2)}$$

Here a figure of 6.36E 8 *cm* was used for σ_g the Earth's radius, and 5.4E 27 *gms* for its mass. Measurement of (A.2) would be somewhat difficult due to the presence of the Sun's gravitational field and those of the nearby planets.
The extent of the Earth's field becomes, from (2.10),

$$\sigma_E = 1.43E17\,cm \qquad \text{(A.3)}$$

which is 8.89E 11 miles, (9658.3 A.U.).

A.3 The Universe.

In Phase I of a Relativistic Domain Universe, both its mass and diameter are changing and so the parameters discussed here will be variable, [5]. In Phase II, the mass will be stable, 1.65E 55 gms, [6], [7], while its radius will be very slowly increasing. In a future paper, it will be shown that the radius of the Universe at the present day is of the order of 1.44E 28 *cm*, (15.1E 9 L.Y.). From (2.9) these numbers give the value of the constant retardation term for the Universe as

$$a_U = 2.39E-6\,cm/sec^2 \qquad \text{(A.4)}$$

This number is considered high and may reflect the fact that the effect described here may not be fully applicable to celestial bodies in which their constituent parts were not physically, (atomically), connected, i.e. the Universe and the galactic bodies contained therein.

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