GRAVITATION - EXPRESSION IN THE FORM OF MAXWELL'S EQUATIONS.

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ABSTRACT.

This paper investigates the derivation of the mathematical formulation of the Relativistic Domain theory of gravitation, in the form of Maxwell's equations of electromagnetic theory.

1 INTRODUCTION.

During the 19^{th} Century, several attempts were made to express the mathematics of gravitation in the form of a field theory. The earliest is perhaps that of Michael Faraday who between the years 1832 to 1859, made a number of experimental searches for a "gravitoelectric" field. He was unsuccessful but remained convinced until he died that such a field existed, [1].
Subsequently, circa 1860 to 1870, James Clerk Maxwell also considered the possibility of expressing gravitation as a field theory, along the lines of his own electromagnetic formulation. However, for like sources he had difficulty reconciling the reversed direction in which the respective forces operate, i.e. like charges repel electrostatically, whereas like masses attract gravitationally. He also had difficulty with the concept of "action at a distance" as implied in Sir Isaac Newton's then extant theory of gravitation. Consequently, he was unable to achieve a satisfactory result, [2].
In the 1890's, Oliver Heaviside also considered this problem. He published a paper, [3], reviewed in [14], in which he showed that the divergence of a static gravitational field obeys Poisson's equation, and for which the curl is zero. He then argued that by analogy to Maxwell's equations, a "gravitomagnetic" field should also exist which had a divergence of zero and a non-zero curl proportional to the velocity of the mass causing this field. Although evidence of such gravitomagnetic effects were not available, Heaviside nevertheless continued the derivation by including time dependent effects of the fields, to produce exact analogies of Maxwell's electromagnetic equations. By this means the propagation velocity of gravitational waves was postulated to be the velocity of light.

All of the above investigations were carried out with Newton's linear theory of gravity which infers "action at a distance", i.e. no intervening medium of transmission. With the advent of Einstein's General Theory of Relativity, the problem became considerably more complex in view of the non-linearity of same. To express this theory in terms of Maxwell type equations, it was necessary to "linearise" Einstein's theory which meant that only weak gravitational sources were being investigated. Nevertheless, the consensus is, [4], [5], [6], [7] and [8], that the existence of a gravitomagnetic field is predicted from this theory, although its effects are likely to be immeasurably small, [8].

In this paper, the gravitational theory to be formulated in the form of Maxwell's equations is that of the Relativistic Domain theory of gravitation as developed in [9], [10] and [11]. This task

is much simpler than that for the General Theory because the Relativistic Domain theory of gravitation is a completely linear theory and is already a field theory. This was briefly demonstrated in [11].
It is very important that to fully appreciate the subject content of this paper, the references [9], [10] and [11] are read first.
Also note that in this paper, vectors and unit vectors are represented by emboldened characters.

2 Relativistic Domain Gravitation in the Form of Maxwell's Equations.

2.1 Preamble.

It has been stated in the literature, i.e. [8] et al, that one of the difficulties in establishing an analogy between electrostatics and gravitation for both Newton's and Einstein's theories, is that electrical charge can take either of two values, +ve or -ve, whereas its proposed gravitation analogy, mass, is always only +ve. As a result, in electrostatics, unlike charges generate an attractive force and like charges generate a repulsive force, whereas, in gravitation, "like masses" result in an attractive force. Thus, for Newton's and Einstein's theories gravitation is, unlike electrostatics, a single valued process and of opposite sense.

This difficulty does not exist with the Relativistic Domain theory of gravitation because, as already shown in [12], under the right conditions, its effect can be reversed to exhibit a repulsive force. This is not however, the result of a reversal of the sign of mass, but rather the reversal of the sign of another parameter. This was demonstrated in [12].

One result of the linear nature of Maxwell's equations, and the linear nature of Newton's theory and of the linearization for weak fields of Einstein's equations, is that the process known as superposition can be employed. This was effectively used by Heaviside in his investigations of the subject. It is via this process that it was predicted in Heaviside's paper, and other literature since, that the velocity of propogation of "gravity waves" is the speed of light. While this process produces sets of analogous equations between two theories of a similar genre, and thereby allows theoretical development of the second along the same lines as the first, it does not however, necessarily mean that the parameters so obtained actually exist. Because of this uncertainty in its use, the principle of superposition will, wherever possible, be avoided in this paper and all derivation be pursued from first principles.

It should also be noted that Maxwell's equations in electromagnetic theory applies to matter on an atomic scale. Velocities and reaction times are therefore very fast. In gravitational theory, the masses involved are that of ponderable matter and thus velocities and reaction times are accordingly very slow.

Finally, it is noted that Maxwell's equations are all spatial in nature and are concerned entirely with effects external to the source. Gravitation on the other hand, is also effective inside the source and has a considerable temporal effect. These additional factors will be considered in the development here.

2.2 A Brief Review of the Maxwell Equations of Electromagnetics.

The development herein will, for the most part, be conducted with the differential form of Maxwell's equations, the most general form of which are as follows,

$$\begin{aligned} \nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \cdot \boldsymbol{D} &= q_v \\ \nabla \times \boldsymbol{H} &= \boldsymbol{J_c} + \frac{\partial \boldsymbol{D}}{\partial t} \\ \nabla \cdot \boldsymbol{B} &= 0 \end{aligned} \tag{2.1}$$

where

$\boldsymbol{E}$ is the Electric field intensity, (gms/coulomb $\equiv$ F/Q).

$\boldsymbol{D} = \varepsilon \boldsymbol{E}$ is the Surface charge density, (coulombs/cm^2 $\equiv$ Q/L^2).

$\boldsymbol{H}$ is the Magnetic flux intensity, (gms/weber = F/W $\equiv$ Q/TL).

$\boldsymbol{B} = \mu \boldsymbol{H}$ is the Magnetic flux density, (webers/cm^2 = W/L^2 $\equiv$ FT/LQ).

$\boldsymbol{J_c}$ is the Conduction current density, (Amps/cm^2 = I/L^2 $\equiv$ Q/TL^2).

q_v is the Volume charge density, (Coulombs/cm^3 $\equiv$ Q/L^3).

ε is the Permittivity of the medium, (Farads/cm $\equiv$ Q^2/L^{2}F).

μ is the Permeability of the medium, (Henry/cm $\equiv$ FT^2/Q^2).

∇ is the usual partial differential operator.

In (2.1), $\partial \boldsymbol{D}/\partial$t is called the displacement current density and the term $\partial \boldsymbol{B}/\partial$t is simply known as the time rate of change of $\boldsymbol{B}$. In the parametric dimensional definitions, Q is charge, F is force, L is length, I is current, W is magnetic flux and T is time.

The objective is now to determine, where they exist, the gravitational analogues of the parameters in (2.1), and from these to construct the equivalent gravitational Maxwell equations.

2.3 Derivation of the Applicable Gravitational Parameter Equivalents.

2.3.1 The Electrostatic Parameters.

Of the two sets of parameters, electrostatic and magnetic, the former are the easier from which to obtain the gravitational analogues. This can be done by starting from the equivalence relationships for the generation of force, i.e. in generalised vector form,

$$\boldsymbol{F_e} = \frac{qQ}{4\pi\varepsilon r^2}\boldsymbol{r} \quad and \quad \boldsymbol{F_g} = -\frac{\gamma m M}{\sigma^2}\boldsymbol{\sigma} \tag{2.2}$$

Where

Q is the electrostatic charge on the source.

q is a test charge of the same parity as Q.

r is the distance separating the charges.

γ is Newton's constant of proportionality.

M is the mass of the gravitational source.

m is a test mass.

σ is the gravitational distance, $(r + \alpha)$, separating the masses, (α is the gravitational radius of the source).

$\boldsymbol{r}$ and $\boldsymbol{\sigma}$ are unit radial vectors.

From (2.2) it is clear that, as stated earlier and in the literature, that the mass of the gravitational source, M, is the analogous parameter to the electrostatic charge, Q.
It is important to note that the constant of proportionality in each case in (2.2) is,
electrostatic

$$k_e = \frac{1}{4\pi\varepsilon} \tag{2.3}$$

gravitational

$$k_g = -\gamma$$

In (2.2), dividing the electric force of repulsion by the test charge yields

$$\frac{\boldsymbol{F_e}}{q} = \frac{Q}{4\pi\varepsilon\, r^2}\boldsymbol{r} = \boldsymbol{E} \tag{2.4}$$

Performing the same operation on the gravitational half of (2.2) gives

$$\frac{\boldsymbol{F_g}}{m} = -\frac{\gamma M}{\sigma^2}\boldsymbol{\sigma} = \boldsymbol{A_\sigma} \tag{2.5}$$

Where clearly $\boldsymbol{A_\sigma}$ is the Acceleration Potential of the gravitational source, see [9], and it is this parameter that may be considered as the gravitational analogy to $\boldsymbol{E}$, the electric field intensity. Accordingly $\boldsymbol{A_\sigma}$ could also be known as the gravitational field intensity.
The electrostatic parameter $\boldsymbol{D}$ is obtained from $\boldsymbol{E}$ by multiplying by ε, the permittivity of the medium, thus

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} \tag{2.6}$$

It is important to note from (2.3) that this means

$$\boldsymbol{D} = \frac{\boldsymbol{E}}{4\pi k_e} \tag{2.7}$$

Performing this operation in the gravitational case yields

$$\boldsymbol{D_\sigma} = \frac{\boldsymbol{A_\sigma}}{4\pi k_g} = \frac{M}{4\pi\sigma^2}\boldsymbol{\sigma} = \rho_s \boldsymbol{s} \tag{2.8}$$

where

ρ_s is the apparent surface mass density of the source at a distance σ from its centre.

$\boldsymbol{s}$ is an area unit vector.

$\boldsymbol{D_\sigma}$ is a new gravitational parameter with dimensions of mass/cm^2 and may accordingly be designated as the surface mass density vector of the gravitational source.
The term -1/4$\pi\gamma$ would thus be analogous to ε and consequently be the "spatial gravitational permittivity" of the medium, say ε_σ.
To show that $\boldsymbol{D_\sigma}$ is indeed the gravitational parameter analogous to $\boldsymbol{D}$, its divergence is determined thus for the situation inside the gravitational source.

$$\nabla_4 \cdot \boldsymbol{D_{i\sigma}} = -\frac{1}{4\pi\gamma}\nabla_4 \cdot \boldsymbol{A_{i\sigma}} \tag{2.9}$$

and in spherical co-ordinates this reduces to

$$\nabla_4 \cdot \boldsymbol{D_{i\sigma}} = -\frac{1}{4\pi\gamma}\left\{\frac{1}{\sigma_i^2}\frac{\partial}{\partial\sigma_i}\left(\sigma_i^2 \boldsymbol{A_{i\sigma}}\right)\right\} \tag{2.10}$$

where

$\boldsymbol{D}_{i\sigma}$ is the value of $\boldsymbol{D}_\sigma$ internal to the source at a distance σ_i from its centre.

$\boldsymbol{A}_{i\sigma}$ is the value of $\boldsymbol{A}_\sigma$ internal to the source at a distance σ_i from its centre.

∇_4 is the four vector equivalent of ∇, (includes the temporal dimension).

From [11] Eq.(3.35), substitution for $\boldsymbol{A_{i\sigma}}$ in (2.10), gives

$$\nabla_4 \cdot \boldsymbol{D_{i\sigma}} = \frac{1}{4\pi\gamma}\left\{\frac{1}{\sigma_i^2}\frac{\partial}{\partial\sigma_i}\left(\frac{\gamma M\sigma_i^3}{\sigma_g^3}\right)\right\} \tag{2.11}$$

which works out to

$$\nabla_4 \cdot \boldsymbol{D_{i\sigma}} = \frac{3M}{4\pi\sigma_g^3} = \rho_v \tag{2.12}$$

where

σ_g is the physical radius of the source.

ρ_v is the volume density of the source.

Thus it is clear from (2.12) that, inside the source, $\boldsymbol{D_{i\sigma}}$ is the analagous gravitational parameter to $\boldsymbol{D}$.

For the situation outside of the gravitational source, it is easily shown by a similar process to that above that

$$\nabla_4 \cdot \boldsymbol{D_\sigma} = 0 \tag{2.13}$$

The analagous parameters to $\boldsymbol{J_c}$ and $\partial\boldsymbol{D}/\partial\tau$ are of much less importance than $\boldsymbol{E_\sigma}$ and $\boldsymbol{D_\sigma}$ but can exist and are briefly considered below.

To determine the analogous gravitational parameter to $\boldsymbol{J_c}$ in (2.1), cognizance is taken of the manner in which $\boldsymbol{J_c}$ is determined in [13], viz.

In a current carrying wire if $n$is the number of electrons per unit length, each with a charge q_e and an average velocity v_e then the charge rate along the wire is

$$\frac{dq}{dt}\boldsymbol{l} = nq_ev_e \tag{2.14}$$

where

$\boldsymbol{l}$ is a unit vector along the wire length.

If the cross sectional area of the wire is a, then

$$\boldsymbol{J_c} = -\frac{nq_e\boldsymbol{v_e}}{a} \tag{2.15}$$

Finally, if this current flow is the result of an electric field, then

$$\boldsymbol{J_c} = \beta\boldsymbol{E} \tag{2.16}$$

where

β is the conductivity of the wire.

The gravitational equivalent of $\boldsymbol{J_c}$ would then be determined as follows.
If a mass m was falling, with a velocity of v_m, under the influence of a gravitational source, and its density was uniform such that its mass per unit length was constant at δ, then the conduction mass flow rate would be

$$\frac{dm}{d\tau}\boldsymbol{\sigma} = \delta \boldsymbol{v_m} \tag{2.17}$$

where

τ is gravitational time.

Thus if the average cross sectional area of the mass were a_m then the conduction mass flow rate density would be

$$\boldsymbol{J_\sigma} = \frac{\delta \boldsymbol{v_m}}{a_m} = \rho_m \boldsymbol{v_m} \tag{2.18}$$

which clearly has dimensions of M/L^{2}T. If (2.16) applies in the gravitational case, then

$$\boldsymbol{J_\sigma} = \beta_\sigma \boldsymbol{A_\sigma} \tag{2.19}$$

and β_σ would be the mass flow conductivity of the medium with dimensions F/TL^2, (the same as β). It has been shown in [10] that external to the source the gravitationally induced velocity of a mass is subject to a maximum value. Therefore it is proposed that the parameter β_σ does exist, and has a value $< \infty$. In this case, under steady state conditions, v_m above would be a maximum value.

The analogy to displacement current is determined as follows. If the gravitational Acceleration Potential changes, this would cause a change in the conduction mass flow rate density. The change in $\boldsymbol{A_\sigma}$ in this case could only be, realistically, a change in the mass of the source, so that from (2.5)

$$\frac{\partial \boldsymbol{A_\sigma}}{\partial \tau} = \frac{\gamma}{\sigma^2}\frac{\partial M}{\partial \tau}\boldsymbol{\sigma} = \frac{\gamma}{\sigma^2}\frac{\partial}{\partial \tau}\left(\frac{4}{3}\pi\sigma_g^3\rho_v\right)\boldsymbol{\sigma} \tag{2.20}$$

This works out to

$$\frac{\partial \boldsymbol{A_\sigma}}{\partial \tau} = 4\pi\gamma\rho_v\frac{\sigma_g^2}{\sigma^2}\frac{\partial \sigma_g}{\partial \tau}\boldsymbol{\sigma} \tag{2.21}$$

So that from (2.8) this becomes

$$\frac{\partial \boldsymbol{D_\sigma}}{\partial \tau} = \rho_v\frac{\sigma_g^2}{\sigma^2}\frac{\partial \sigma_g}{\partial \tau}\boldsymbol{\sigma} \tag{2.22}$$

Consequently $\partial\sigma_g/\partial\tau$ is the rate at which the radius of the gravitational source changes to cause a variation in M and thereby $\boldsymbol{A_\sigma}$. Eq.(2.22) is therefore the mass flow rate density due to a variation in the mass of the gravitational source. The total mass flow rate density then becomes

$$\boldsymbol{J_g} = \boldsymbol{J_\sigma} + \boldsymbol{J_D} = \left(\rho_m v_m + \rho_v\frac{\sigma_g^2}{\sigma^2}\frac{\partial \sigma_g}{\partial \tau}\right)\boldsymbol{\sigma} \tag{2.23}$$

and which from (2.19) and (2.22) can be written

$$\boldsymbol{J_g} = \boldsymbol{J_\sigma} + \frac{\partial \boldsymbol{D_\sigma}}{\partial \tau} \tag{2.24}$$

These effects and their relationships can only occur external to the source. They describe the dynamic results of gravitation on other material bodies and the effects of physical change to the source itself. They are therefore not directly associated with the generation of the gravity fields, and, due to the results of the following Section, do not appear in the Maxwell gravitic equations.

This completes the identification of the gravitational analogies to the electrostatic parameters. Note that they could all have been obtained via the superposition principle which provides corroborative support to their derivation.

2.3.2 ***The Analogous Magnetic Parameters, Spatial.***

The difficulty in identifying these parameters, if they exist, is threefold. Firstly, unlike the electrostatic case, there is no source from which an analogy can be drawn, i.e. a magnetic field does not emanate from a magnetic source but from the motion of electric charge. Secondly, the precise mechanism that produces a magnetic flux from the flow rate of charge is not known. Finally, as stated earlier from [8], any effect on ponderable matter is likely to be vanishingly small. Consequently, the effect of such a field has never been measured. Therefore, to assess whether such a field exists, it is necessary to qualitatively estimate the likely effect in an astronomically realistic situation. To do so requires the assumption that it is produced by the motion of the gravitational source. It also requires the assumption that the analogous parameter to μ, permeability, also exists. Such a parameter, μ_σ, would by the principle of superposition, have dimensions of L/M, (FT^2/M^2), so that with the analogous parameter to $\boldsymbol{H}$, say $\boldsymbol{H_\sigma}$, having dimensions of M/TL, then the analogous parameter to $\boldsymbol{B}$, $\boldsymbol{B_\sigma}$, would be given by

$$\boldsymbol{B_\sigma} = \mu_\sigma \boldsymbol{H_\sigma} \tag{2.25}$$

and then the dimensions of $\boldsymbol{B_\sigma}$ would be inverse time.

Assume a large gravitational source, such as a star, was in motion such that a $\boldsymbol{B_\sigma}$ field was generated. Now, any orbiting planetary mass possessing a spatial orbital velocity component v_{or} that was parallel to the $\boldsymbol{B_\sigma}$ field would then be subject to a force, transverse to the direction of motion of the source, given by

$$\boldsymbol{F_{or}} = m\boldsymbol{v_{or}} \cdot \boldsymbol{B_\sigma} \tag{2.26}$$

An astronomical gravitational source can possess two forms of motion, a translational motion due to the rotation of its parent galaxy, and a rotational motion due to its own spin. Consider first the translational motion, this would result in a $\boldsymbol{B_{T\sigma}}$ field as shown below.

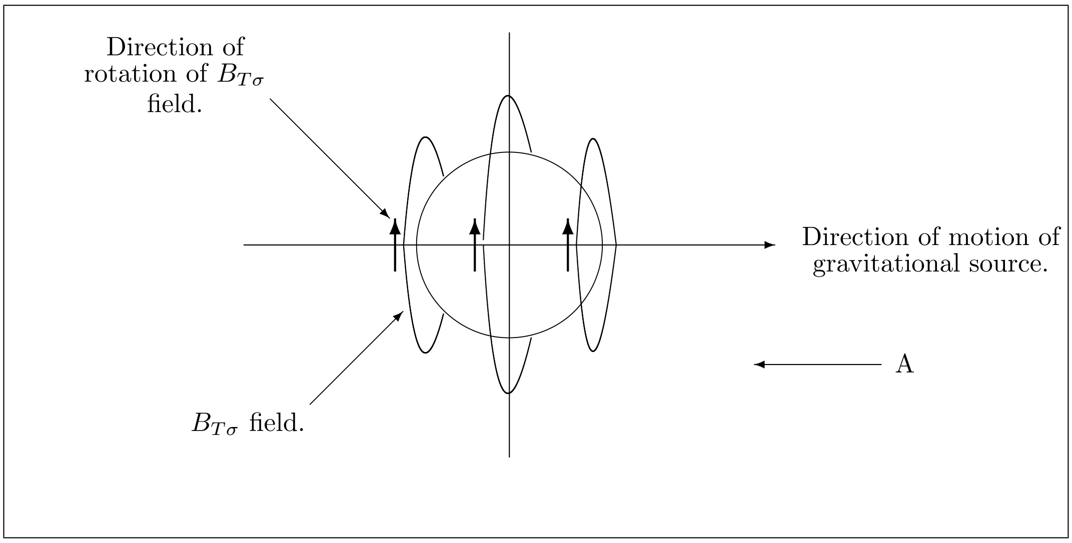

Fig. 2.1 Generation of the $\boldsymbol{B_{T\sigma}}$ Field.

The spin motion would accordingly produce a toroidal field, $\boldsymbol{B_{R\sigma}}$ thus,

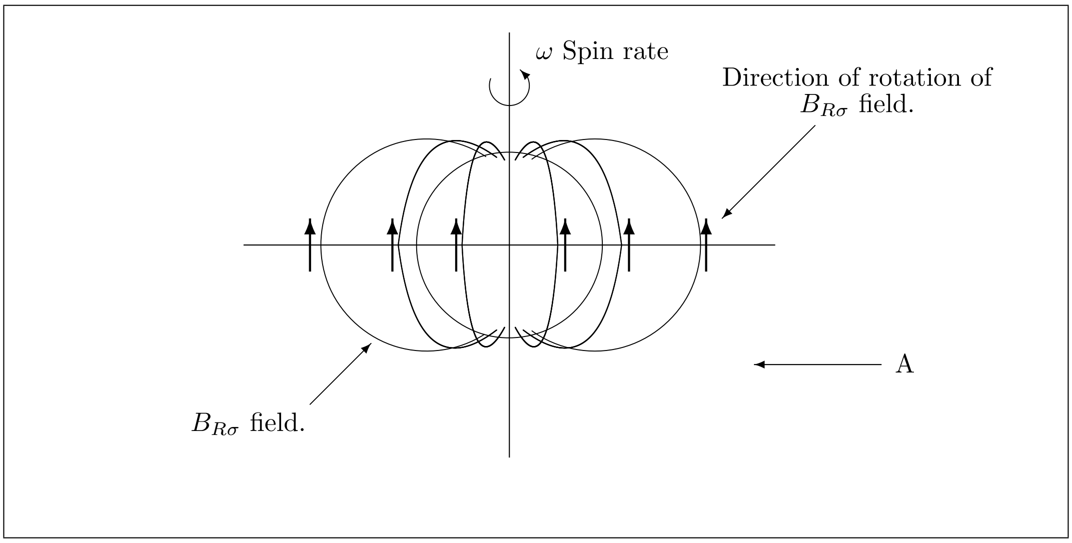

Fig. 2.2 Generation of the $\boldsymbol{B_{R\sigma}}$ Field.

In Figs. 2.1 and 2.2, looking in direction A, the combination of the two fields would then take the form as shown, (approximately), in Fig. 2.3 below. On the LHS the fields would be additive, while on the RHS, subtractive. Thus, according to (2.26), the orbital plane of any planetary mass in such a field would, over a very long period of time, precess in the same direction as the field. When on the RHS its direction of precession would depend upon the relative strengths of $\boldsymbol{B_{T\sigma}}$ and $\boldsymbol{B_{R\sigma}}$. If $\boldsymbol{B_{T\sigma}} > \boldsymbol{B_{R\sigma}}$, (as shown in Fig.2.3), the planetary orbit would continuously rotate completely around the star. If $\boldsymbol{B_{R\sigma}} > \boldsymbol{B_{T\sigma}}$ then the planetary orbit plane would eventually stabilise parallel to the spin axis of the star.

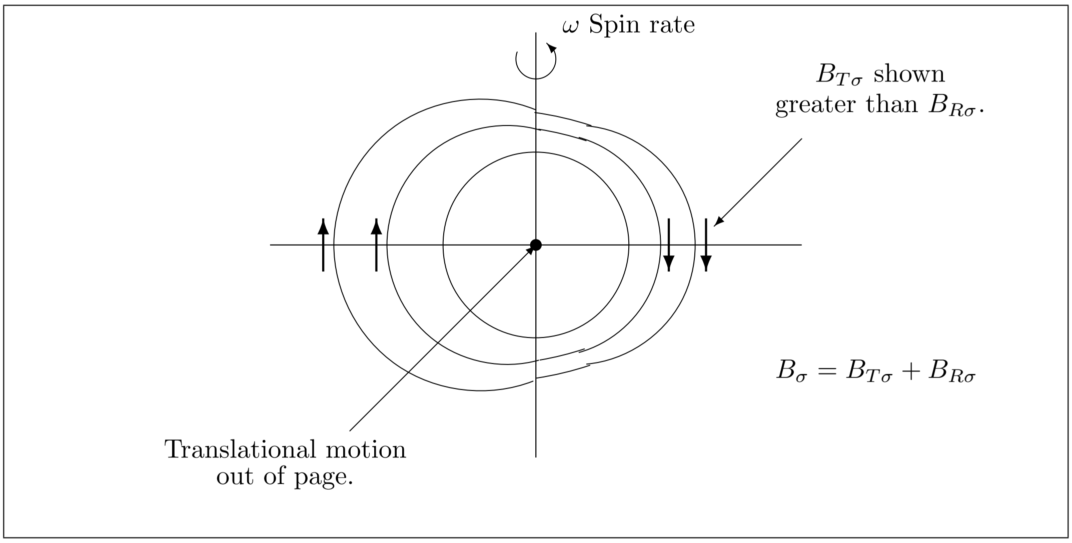

Fig. 2.3 Composite $\boldsymbol{B_\sigma}$ Field due to Combined Translational and Spin Motions.

Inside the star the same fields would exist but would now cause the precession of the gaseous material of the star itself. This would have two effects. Firstly, the precessional motion of internal matter would itself set up a $\boldsymbol{B_{iR\sigma}}$ field opposite to the direction of the spin which would tend to effectively cause a slight reduction in the spin motion. The second effect would, because of the slower rate of material precession on the RHS of the star in Fig.2.3, there would be a build up of density on the RHS which would then cause a slight nutation of the spin axis, and also induce a small translational motion wobble.
Observation of the Sun and the orbital planes of the planets of the Solar System, indicate that none of the above effects have become apparent over the five or so billion years that the system is thought to have been in existence. Furthermore, observation of the proper motion of the nearby stars in the galaxy, similarly do not indicate an erratic motion as they orbit its centre.
Therefore, from the above dissertation, the final conclusion reached here is that analogous gravitational parameters to $\boldsymbol{B}$ and $\boldsymbol{H}$ do not exist. A possible explanation for this is presented in Section 3.

2.4 Derivation of the Applicable Gravitational Equivalents - Temporal.

It is shown in Appendix B that external to the source, the variation of temporal velocity, i.e. the Acceleration Potential in the temporal direction is given by

$$\boldsymbol{A_\tau} = -\frac{\left(1-u^2\right)^{1/2}}{u}\left|\boldsymbol{A_\sigma}\right| \boldsymbol{x_0} \tag{2.27}$$

where

$|\boldsymbol{A}_\sigma|$ is the magnitude of $\boldsymbol{A_\sigma}$.

$\boldsymbol{x}_0$ is a unit vector in the temporal direction.

Eq.(2.27) is therefore the gravitational equivalent of $\boldsymbol{E}$ in the temporal direction.
To determine the gravitational equivalent of $\boldsymbol{D}$ in that dimension, note that it would take the form

$$\boldsymbol{D_\tau} = \varepsilon_\tau \boldsymbol{A_\tau} \tag{2.28}$$

and would, similar to (2.8) possess dimensions of mass/cm^2. Accordingly its divergence would possess dimensions of mass/cm^3. Clearly these parameters can only be the mass surface density and mass volume density and as such must be invariant. Consequently, from (2.27) and (2.28) it must therefore mean that

$$\varepsilon_\tau = -\frac{u}{(1-u^2)^{1/2}}\varepsilon_\sigma \tag{2.29}$$

so that from (2.28) and (2.29)

$$\boldsymbol{D_\tau} = -\frac{u}{(1-u^2)^{1/2}}\varepsilon_\sigma \boldsymbol{A_\tau} = \frac{|\boldsymbol{A_\sigma}|}{4\pi\gamma} = \rho_s\, \boldsymbol{x_0} \tag{2.30}$$

Therefore it is clear that

$$\begin{aligned} |\boldsymbol{D_\tau}| = |\boldsymbol{D_\sigma}| &= \rho_s \quad (internal) \\ &= 0 \quad (external) \end{aligned} \tag{2.31}$$

and

$$\begin{aligned} \nabla_4 \cdot \boldsymbol{D_\tau} = \nabla_4 \cdot \boldsymbol{D_\sigma} &= \rho_v \quad (internal) \\ &= 0 \quad (external) \end{aligned}$$

Eq.(2.29) cannot be proved theoretically but, will be shown in Appendix C to be correct via the solutions to the Maxwell equations that result from the above.
The non-existence of the spatial fields $\boldsymbol{H_\sigma}$ etc necessarily determines their non-existence in the temporal direction.

2.5 The Gravitational Maxwell Equations.

In Appendix B the curl of both the spatial and temporal versions of the Acceleration Potential, both internal and external to the source are shown to be zero, i.e. all Acceleration Potentials are irrotational, and consequently, from this and the preceding derivations, the gravitational Maxwell equations can now be stated.
There are four sets of equations required to fully express gravitation in the form of Maxwell's equations. They are as follows.
(i) Spatial, external to the source.

$$\begin{aligned} \nabla_4 \times \boldsymbol{A_\sigma} &= 0 \\ \nabla_4 \cdot \boldsymbol{D_\sigma} &= 0 \end{aligned} \tag{2.32}$$

(ii) Spatial, internal to the source.

$$\begin{aligned} \nabla_4 \times \boldsymbol{A_\sigma} &= 0 \\ \nabla_4 \cdot \boldsymbol{D_\sigma} &= \rho_v \end{aligned} \tag{2.33}$$

(iii) Temporal, external to the source.

$$\begin{aligned} \nabla_4 \times \boldsymbol{A_\tau} &= 0 \\ \nabla_4 \cdot \boldsymbol{D_\tau} &= 0 \end{aligned} \tag{2.34}$$

(iv) Temporal, internal to the source

$$\begin{aligned} \nabla_4 \times \boldsymbol{A_\tau} &= 0 \\ \nabla_4 \cdot \boldsymbol{D_\tau} &= \rho_v \end{aligned} \tag{2.35}$$

Eqs (2.32) to (2.35) represent the simplest mathematical formulation for Relativistic Domain gravitation and it is not considered useful to attempt a generalisation in view of this. Their solutions are presented in Appendix C.
The parameters $\boldsymbol{J_\sigma}$ and $\partial\boldsymbol{D_\sigma}/\partial\tau$ do not appear in these equations due to the non-existence of $\boldsymbol{H_\sigma}$ etc and in any case are more associated with the dynamic effects of gravitation than with the generation of the gravity fields.

3 Conclusions.

The most significant point to emerge from this development is the non-existence of the $\boldsymbol{H_\sigma}$ and $\boldsymbol{B_\sigma}$ fields, the gravitational equivalents of the magnetic flux intensity and the magnetic flux density. It is proposed that the reason for this is as follows.

It is believed that there are only three basic fundamental parameters that govern the existence of the entire Universe and everything in it. They are (i) a four dimensional space, one of which is temporal in nature with the other three spatial, (ii) energy, contained within the three spatial dimensions, and (iii) a velocity parameter, c, equal to the velocity of light, to which all existence is subject. All of these parameters are totally invariant. Time is not a fundamental parameter because, as shown by the dilatation effect, its rate of passage is subject to variation in the presence of energy, and to the spatial motion of that energy, as shown respectively in [11] and [15]. Thus time is merely a consequence of the relative motion of, and in, the temporal and the three spatial dimensions.
It is also believed that the spatial motion of all things of whatever nature, mechanical, electrical, atomic et al, must involve a transference of energy from one location to another. Electrostatic charge by itself does not hold energy, and when in motion therefore cannot convey it. This is therefore effected by the generation, via the motion of the charge by a mechanism as yet unknown, of the orthogonal magnetic field. Consequently the electrostatic and magnetic fields associated with an electromagnetic wave, can be considered as the orthogonal components of an energy field, moving in the third direction as exemplified by the Poynting vector. The velocity of propogation is that of light because of the absence of physical mass.
In gravitation, the primary parameter is mass, which, as of course is well known, already contains a vast amount of energy in line with Einstein's mass-energy relationship. Consequently, the movement of a mass automatically involves the transference of energy, and thereby does not necessitate the generation of a secondary parametric field to effect same. Hence the absence of the $\boldsymbol{H_\sigma}$ and $\boldsymbol{B_\sigma}$ fields in the Maxwell versions of gravitic mathematical formulation.

As a result, this form of representation of gravity is consequently somewhat limited. Firstly, it does not provide any insight into the physics involved in the generation of the field. Secondly, it provides no insight into the velocity of propogation of gravitic variations. Thirdly, its solutions, as shown in Appendix C, do not provide any information on the dynamic effects on other bodies, but only on the nature of the generating mechanism. However, this is perhaps as should be expected. Finally, when a physical translational or rotational motion of the source is present, both of which appear as extra boundary conditions in the solution to the temporal versions of the equations, the result could give rise to mis-interpretation, especially if these motions are variable, as neither of these motions will effect the generation of the gravity field, apart from a secondary one, the relativistic increase in mass of the source.
On the other hand, this form of mathematical representation of gravity does provide a simpler and more consistent developmental analysis than that presented in [11], especially so in the temporal direction which was not fully covered in [11]. Accordingly while it brings nothing new to the theory,

it does provides good corroborative support.

APPENDIX A

Inter-Relationship Between the Spatial and Temporal Expansion Velocities and Their Acceleration Potentials.

For simplicity, in this Appendix only vector magnitudes of all parameters are considered.

In [11], Section 3.3.2, relationships for the velocities of spatial expansion, internal and external, emanating from a gravitational source, and the corresponding temporal velocities were developed. These are repeated below for convenience.

(i) Spatial, external.

$$\upsilon_\sigma = \left(\frac{2\gamma M}{\sigma}\right)^{1/2} = c\left(1-u^2\right)^{1/2} \tag{A.1}$$

(ii) Spatial, internal.

$$\upsilon_{i\sigma} = \left(\frac{3\gamma M}{\sigma_g} - \frac{\gamma M \sigma_i^2}{\sigma_g^3}\right)^{1/2} = c\left(1-u_i^2\right)^{1/2} \tag{A.2}$$

(iii) Temporal, external.

$$\upsilon_\tau = c\left(1 - \frac{2\gamma M}{c^2\sigma}\right)^{1/2} = cu \tag{A.3}$$

(iv) Temporal, internal.

$$\upsilon_{i\tau} = c\left(1 - \frac{3\gamma M}{c^2\sigma_g} + \frac{\gamma M \sigma_i^2}{c^2\sigma_g^3}\right)^{1/2} = cu_i \tag{A.4}$$

From these relationships it is clear that

$$V = \left(\upsilon_\sigma^2 + \upsilon_\tau^2\right)^{1/2} = c \quad and \quad V_i = \left(\upsilon_{i\sigma}^2 + \upsilon_{i\tau}^2\right)^{1/2} = c \tag{A.5}$$

and all terms are defined thus

υ_σ the spatial expansion velocity of the source external to itself.

$\upsilon_{i\sigma}$ the spatial expansion velocity of the source internal to itself.

υ_τ the temporal velocity of all spatial points external to the source.

$\upsilon_{i\tau}$ the temporal velocity of all spatial points internal to the source.

γ Newton's constant of proportionality.

M the mass of the gravitational source.

σ the distance of a point external to the gravitational source from its centre.

u the temporal rate at σ.

σ_g the external radius of the gravitational source.

σ_i the distance of a point internal to the gravitational source from its centre.

u_i the temporal rate at σ_i.

V the spatial-temporal velocity magnitude of existence external to the source.

V_i the spatial-temporal velocity magnitude of existence internal to the source.

It was also shown in [11] that

$$u = \left(1 - \frac{2\gamma M}{c^2\sigma}\right)^{1/2} \tag{A.6}$$

and

$$u_i = \left(1 - \frac{3\gamma M}{c^2\sigma_g} + \frac{\gamma M\sigma_i^2}{c^2\sigma_g^3}\right)^{1/2} \tag{A.7}$$

Finally, it was also shown in [11] that

$$A_\sigma = -\frac{\gamma M}{\sigma^2} = -c^2 u\frac{du}{d\sigma} \tag{A.8}$$

and

$$A_{i\sigma} = -\frac{\gamma M\sigma_i}{\sigma_g^3} = -c^2 u_i\frac{du_i}{d\sigma_i} \tag{A.9}$$

This Appendix will now derive the associated acceleration parameters in the temporal direction and establish their relationship with their spatial counterparts.

From (A.3) above, allowing σ to vary

$$A_\tau = \frac{d\upsilon_\tau}{d\tau} = c\frac{d}{d\sigma}\left(1 - \frac{2\gamma M}{c^2\sigma}\right)^{1/2}\frac{d\sigma}{d\tau} \tag{A.10}$$

where $d\sigma/d\tau$ is given by (A.1). Eq.(A.10) works out to be

$$\begin{aligned} A_\tau &= \frac{\left(\frac{2\gamma M}{\sigma}\right)^{1/2}\frac{\gamma M}{\sigma^2}}{c\left(1 - \frac{2\gamma M}{c^2\sigma}\right)^{1/2}} \\ &= c^2\left(1 - u^2\right)^{1/2}\frac{du}{d\sigma} = -\frac{\left(1 - u^2\right)^{1/2}}{u}A_\sigma \end{aligned} \tag{A.11}$$

and similarly from (A.4)

$$A_{i\tau} = \frac{\left(\frac{3\gamma M}{\sigma_g} - \frac{\gamma M \sigma_i^2}{\sigma_g^3}\right)^{1/2} \frac{\gamma M \sigma_i}{\sigma_g^3}}{c\left(1 - \frac{3\gamma M}{c^2\sigma_g} + \frac{\gamma M \sigma_i^2}{c^2\sigma_g^3}\right)^{1/2}}$$

$$= c^2\left(1 - u_i^2\right)^{1/2} \frac{du_i}{d\sigma_i} = -\frac{\left(1 - u_i^2\right)^{1/2}}{u_i} A_{i\sigma} \tag{A.12}$$

Consequently, the spatial-temporal Acceleration Potential vector magnitude is, from (A.5) and (A.8), given by

$$A = \left(A_\sigma^2 + A_\tau^2\right)^{1/2} = \frac{\gamma M}{\sigma^2\left(1 - \frac{2\gamma M}{c^2\sigma}\right)^{1/2}}$$

$$= \frac{A_\sigma}{u} = \frac{A_\tau}{\left(1 - u^2\right)^{1/2}} = c^2 \frac{du}{d\sigma} \tag{A.13}$$

and from (A.6) and (A.9)

$$A_i = \left(A_{i\sigma}^2 + A_{i\tau}^2\right)^{1/2} = \frac{\gamma M}{\sigma_i^2\left(1 - \frac{3\gamma M}{c^2\sigma_g} + \frac{\gamma M \sigma_i}{c^2\sigma_g^3}\right)^{1/2}}$$

$$= \frac{A_{i\sigma}}{u_i} = \frac{A_{i\tau}}{\left(1 - u_i^2\right)^{1/2}} = c^2 \frac{du_i}{d\sigma_i} \tag{A.14}$$

The four dimensional geometrical representation of these parameters can then be shown as

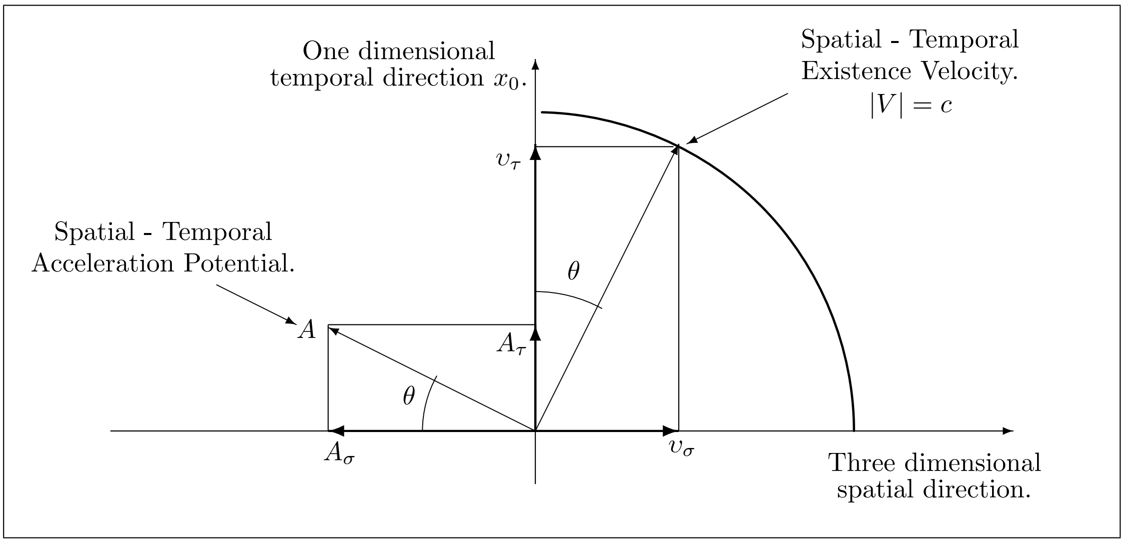

Fig. A.1 Geometrical Representation of Existence Velocities and Acceleration Potentials at Some Point External to the Source.

A similar representation exists internal to the source. Note that the spatial-temporal "angle of existence" is given by

$$\cos\theta = \frac{\upsilon_\tau}{c} = \left(1 - \frac{\upsilon_\sigma^2}{c^2}\right)^{1/2} = \frac{A_\sigma}{A} = u \tag{A.15}$$

and is the same as that derived in [15] for the physical velocity of a mass in Pseudo - Euclidean Space Time.

APPENDIX B.

Derivation of the 4 Vector Curl of the Acceleration Potentials.

The derivations below are for the Acceleration Potentials external to the source. Derivations for the Potentials internal to the source follow the same process with the same results.

B.1 Curl of the Spatial Acceleration Potential Vector.
The spatial Acceleration Potential vector, $\boldsymbol{A_\sigma}$, has only one component, in the σ direction, and therefore by the definition of curl, the 4-vector curl of $\boldsymbol{A_\sigma}$, is zero, i.e.

$$\nabla_4 \times \boldsymbol{A_\sigma} = 0 \tag{B.1}$$

B.2 Curl of the Temporal Acceleration Potential Vector.
The temporal Acceleration Potential vector $\boldsymbol{A_\tau}$, has only one component, in the $\boldsymbol{x}_0$ direction and therefore by the definition of curl, its 4-vector curl is zero. However, because $\boldsymbol{A_\tau}$, is a function of σ, it is necessary to prove this.

$$\begin{aligned}\nabla_4 \times \boldsymbol{A_\tau} &= \nabla_4 \times \left\{ -\frac{\left(1-u^2\right)^{1/2}}{u} \boldsymbol{A_\sigma} \right\} \\ &= -\frac{\left(1-u^2\right)^{1/2}}{u} \nabla_4 \times \boldsymbol{A_\sigma} - \nabla_4 \left\{ \frac{\left(1-u^2\right)^{1/2}}{u} \right\} \times \boldsymbol{A_\sigma}\end{aligned} \tag{B.2}$$

By (B.1) this reduces to

$$\nabla_4 \times \boldsymbol{A_\tau} = -\nabla_4 \left\{ \frac{\left(1-u^2\right)^{1/2}}{u} \right\} \times \boldsymbol{A_\sigma} \tag{B.3}$$

Because u is only a function of σ, the gradient in (B.3) can be written

$$\nabla_4 \left\{ \frac{\left(1-u^2\right)^{1/2}}{u} \right\} = \frac{\partial}{\partial\sigma} \left\{ \frac{\left(1-u^2\right)^{1/2}}{u} \right\} \boldsymbol{\sigma} \tag{B.4}$$

Substituting for u and carrying out the partial differentiation gives

$$\nabla_4 \left\{ \frac{\left(1-u^2\right)^{1/2}}{u} \right\} = -\left\{ \frac{\gamma M}{\sigma^2 \left(\frac{2\gamma M}{c^2\sigma}\right)^{1/2} \left(1 - \frac{2\gamma M}{c^2\sigma}\right)^{3/2}} \right\} \boldsymbol{\sigma} \tag{B.5}$$

and therefore

$$\nabla_4 \times \boldsymbol{A_\sigma} = \left\{ \frac{\gamma M}{\sigma^2 \left(\frac{2\gamma M}{c^2\sigma}\right)^{1/2} \left(1 - \frac{2\gamma M}{c^2\sigma}\right)^{3/2}} \right\} \boldsymbol{\sigma} \times \left(-\frac{\gamma M}{\sigma^2}\right) \boldsymbol{\sigma} = 0 \tag{B.6}$$

APPENDIX C.

Solutions of the Gravitational Maxwell Equations.

It was shown in [11], that gravitation was proportional to the spatial gradient of the linear expansion velocity of space, produced by the gravitational source. Appendix B to this paper, has now completed the derivation of the corresponding temporal effects. Solutions to the gravitational Maxwell equations presented here, will now derive these mathematical representations, covering both the acceleration and velocity parameters, without recourse to the information in the preceding main text and Appendices other than the use of the proportionality parameters, ε_σ and ε_τ.

A total of four solutions are required, one each for each pair of equations (2.32) to (2.35), covering both the spatial and temporal cases both external and internal to the source. Some of the spatial solutions are well known in the literature but are included here both for completeness and because they lead on to the solutions in the temporal domain.

(i) Case 1 - Static, Spatial, External.

In (2.32), because the curl of $\boldsymbol{A_\sigma}$ is zero it can be represented by the gradient of a scalar field thus

$$\boldsymbol{A_\sigma} = \nabla_4 U_\sigma \boldsymbol{\sigma} \tag{C.1}$$

where

U_σ is a scalar field.

Substitution of (C.1) into the second part of (2.32) together with (2.8) gives

$$\nabla_4^2 U_\sigma = 0 \tag{C.2}$$

which is a four dimensional version of Laplace's equation. This, in expanded form is

$$\frac{\partial^2 U_\sigma}{\partial x_0^2} + \frac{1}{\sigma^2}\frac{\partial}{\partial \sigma}\left(\sigma^2 \frac{\partial U_\sigma}{\partial \sigma}\right) = 0 \tag{C.3}$$

Here the spatial part has been expressed in spherical co-ordinates while the temporal part is in Cartesian. This is permitted in view of the special co-ordinate relationship between the spatial and temporal domains. In any case the temporal part is zero because there is no direct variation of gravity with time, $(\partial x_0 = cu\partial\tau)$.

Eq.(C.3) therefore becomes simply

$$\frac{1}{\sigma^2}\frac{\partial}{\partial \sigma}\left(\sigma^2 \frac{\partial U_\sigma}{\partial \sigma}\right) = 0 \tag{C.4}$$

and (C.4) has the solution

$$U_\sigma = -\frac{k_1}{\sigma} + k_2 \tag{C.5}$$

Determination of the constants of integration is as follows. When $\sigma \to \infty$, the gravitational effect tends to zero, whence $U_\sigma \to 0$. Therefore $k_2 = 0$ and (C.5) becomes simply

$$U_\sigma = -\frac{k_1}{\sigma} \tag{C.6}$$

So that, for a spherically symmetric mass, from (C.1)

$$\boldsymbol{A_\sigma} = \frac{\partial U_\sigma}{\partial \sigma}\boldsymbol{\sigma} = \frac{k_1}{\sigma^2}\boldsymbol{\sigma} \tag{C.7}$$

To determine k_1, apply Gauss' law to $\boldsymbol{D_\sigma}$ thus, using (2.8)

$$\oiint \boldsymbol{D_\sigma} \cdot d\boldsymbol{s} = -\frac{\boldsymbol{A_\sigma}}{4\pi\gamma} \oiint d\boldsymbol{s} = M \tag{C.8}$$

From which

$$\boldsymbol{A_\sigma} = -\frac{\gamma M}{\sigma^2}\boldsymbol{\sigma} \tag{C.9}$$

i.e. as at (2.5). Thus from (C.7) and (C.9)

$$k_1 = -\gamma M \tag{C.10}$$

So that in (C.6)

$$U_\sigma = \frac{\gamma M}{\sigma} \tag{C.11}$$

U_σ has the dimensions of a velocity squared and can be related to the spatial expansion velocity of the source as follows

$$\boldsymbol{A_\sigma} = \frac{d\boldsymbol{\upsilon_\sigma}}{d\tau} = \upsilon_\sigma \frac{d\boldsymbol{\upsilon_\sigma}}{d\sigma} = \frac{dU_\sigma}{d\sigma}\boldsymbol{\sigma} \tag{C.12}$$

From which

$$U_\sigma = \frac{\upsilon_\sigma^2}{2} \tag{C.13}$$

and therefore from (C.11)

$$\upsilon_\sigma = \left(\frac{2\gamma M}{\sigma}\right)^{1/2}\boldsymbol{\sigma} = c\left(1 - u^2\right)^{1/2}\boldsymbol{\sigma} \tag{C.14}$$

as stated in Appendix A et al and derived in [11]. U_σ is clearly, from [11], Eq.(3.38) the Newtonian Potential, (spatial), for the Relativistic Domain theory of gravitation external to the source.

(ii) Case 2 - Static, Spatial, Internal.

To solve (2.33) note that, for this case, (C.2) becomes

$$\nabla_4^2 U_{i\sigma} = -4\pi\gamma\rho_v \tag{C.15}$$

and thus (C.4) becomes, (the temporal component is again zero)

$$\frac{1}{\sigma_i^2}\frac{\partial}{\partial \sigma_i}\left(\sigma_i^2 \frac{\partial U_{i\sigma}}{\partial \sigma_i}\right) = -4\pi\gamma\rho_v = -\frac{3\gamma M}{\sigma_g^3} \tag{C.16}$$

This has the solution

$$U_{i\sigma} = -\frac{\gamma M \sigma_i^2}{2\sigma_g^3} - \frac{k_1}{\sigma_i} + k_2 \tag{C.17}$$

To determine k_2 note that when $\sigma_i = \sigma_g$, from (C.11), $U_{i\sigma} = U_g = \gamma M/\sigma_g$ so that in (C17) this gives

$$k_2 = \frac{3\gamma M}{2\sigma_g} + \frac{k_1}{\sigma_g} \tag{C.18}$$

which gives in (C.17)

$$U_{i\sigma} = -\frac{\gamma M\sigma_i^2}{2\sigma_g^3} - \frac{k_1}{\sigma_i} + \frac{3\gamma M}{2\sigma_g} + \frac{k_1}{\sigma_g} \tag{C.19}$$

Thus for a spherically symmetric mass, at σ_i

$$\boldsymbol{A_{i\sigma}} = \frac{\partial U_{i\sigma}}{\partial\sigma_i} = \left(-\frac{\gamma M\sigma_i}{\sigma_g^3} + \frac{k_1}{\sigma_i^2}\right)\boldsymbol{\sigma} \tag{C.20}$$

Now, applying Gauss' law to $\boldsymbol{D}_{i\sigma}$

$$\oiint \boldsymbol{D_{i\sigma}} \cdot d\boldsymbol{s_i} = -\frac{\boldsymbol{A_{i\sigma}}}{4\pi\gamma}\oiint d\boldsymbol{s_i} = M_i = M\frac{\sigma_i^3}{\sigma_g^3} \tag{C.21}$$

where

- s_i is the surface area of a sphere of radius σ_i inside the source.
- M_i is the mass of the source contained within σ_i.

From (C.21)

$$\boldsymbol{A_{i\sigma}} = -\frac{\gamma M\sigma_i}{\sigma_g^3}\boldsymbol{\sigma} \tag{C.22}$$

as derived in [11], and equating this to (C.20) shows that $k_1 = 0$ so that in (C.19)

$$U_{i\sigma} = -\frac{\gamma M\sigma_i^2}{2\sigma_g^3} + \frac{3\gamma M}{2\sigma_g} \tag{C.23}$$

From [11], Eq.(3.46) $U_{i\sigma}$ is therefore the Newtonian Potential inside the source at a distance σ_i from the centre. In an identical process for the external case it is seen that it is related to the internal spatial expansion velocity thus

$$U_{i\sigma} = \frac{\upsilon_{i\sigma}^2}{2} \tag{C.24}$$

so that from (C.23) and (C.24)

$$\upsilon_{i\sigma} = \left(\frac{3\gamma M}{\sigma_g} - \frac{\gamma M\sigma_i^2}{\sigma_g^3}\right)^{1/2}\boldsymbol{\sigma} = c\left(1-u_i^2\right)^{1/2}\boldsymbol{\sigma} \tag{C.25}$$

as derived in [11] and stated in Appendix A.

(iii) Case 3 - Static, Temporal, External.
Solution of (2.34) in the temporal domain is a little more involved. In this case the equivalent of (C.2) is

$$\boldsymbol{A_\tau} = \nabla_4 U_\tau \boldsymbol{x_0} \tag{C.26}$$

So that from (2.30)

$$-\frac{\left(1-u^2\right)^{1/2}}{u\varepsilon_\sigma}\boldsymbol{D_\tau} = \nabla_4 U_\tau \boldsymbol{x_0} \tag{C.27}$$

and thus

$$\boldsymbol{D_\tau} = \frac{u}{4\pi\gamma\left(1-u^2\right)^{1/2}}\nabla_4 U_\tau \boldsymbol{x_0} \tag{C.28}$$

Taking the divergence, from (2.31)

$$\nabla_4 \cdot \boldsymbol{D_\tau} = \nabla_4 \cdot \left(\frac{u}{4\pi\gamma\left(1-u^2\right)^{1/2}}\nabla_4 U_\tau \boldsymbol{x_0}\right) = 0 \tag{C.29}$$

Because $|\boldsymbol{D_\tau}| = |\boldsymbol{D_\sigma}|$, (C.29) can be written thus

$$\frac{1}{4\pi\gamma}\frac{\partial}{\partial\sigma}\left(\frac{\sigma^2 u}{\left(1-u^2\right)^{1/2}}\nabla_4 U_\tau\right) = 0 \tag{C.30}$$

Taking the first integral

$$\nabla_4 U_\tau = \frac{k_1}{\sigma^2}\frac{\left(1-u^2\right)^{1/2}}{u}\boldsymbol{x_0} = \boldsymbol{A_\tau} \tag{C.31}$$

$\boldsymbol{A_\tau}$ is the temporal Acceleration Potential vector and therefore lies along the $\boldsymbol{x}_0$ axis only. The gradient of U_τ can therefore be written

$$\frac{\partial U_\tau}{\partial x_0} = \frac{k_1}{\sigma^2}\frac{\left(1-u^2\right)^{1/2}}{u}\boldsymbol{x_0} \tag{C.32}$$

and with $\partial x_0 = cu\partial\tau$ this becomes

$$\frac{1}{cu}\frac{\partial U_\tau}{\partial\sigma}\frac{\partial\sigma}{\partial\tau} = \frac{k_1}{\sigma^2}\frac{\left(1-u^2\right)^{1/2}}{u}\boldsymbol{x_0} \tag{C.33}$$

Inserting (C.14) for $\partial\sigma/\partial\tau$ this becomes

$$\frac{\partial U_\tau}{\partial\sigma} = \frac{k_1}{\sigma^2}\boldsymbol{x_0} \tag{C.34}$$

Integrating

$$U_\tau = -\frac{k_1}{\sigma} + k_2 \tag{C.35}$$

Exactly the same generic result as in the spatial domain.
To determine k_2 note that when $\sigma \to \infty$ the temporal velocity, $\upsilon_\tau \to c$ so that via similarity with the spatial domain

$$U_\tau|_{\sigma\to\infty} = \left.\frac{\upsilon_\tau^2}{2}\right|_{\sigma\to\infty} = \frac{c^2}{2} = k_2 \tag{C.36}$$

This gives in (C.35)

$$U_\tau = \frac{c^2}{2} - \frac{k_1}{\sigma} \tag{C.37}$$

Now apply Gauss' law to $\boldsymbol{D_\tau}$ thus

$$\oint\!\!\!\oint \boldsymbol{D_\tau}\cdot d\boldsymbol{s} = \frac{u}{\left(1-u^2\right)^{1/2}}\frac{\boldsymbol{A_\tau}}{4\pi\gamma}\oint\!\!\!\oint d\boldsymbol{s} = M \tag{C.38}$$

From this it is seen that

$$\boldsymbol{A_\tau} = \frac{\left(1-u^2\right)^{1/2}}{u}\frac{\gamma M}{\sigma^2}\boldsymbol{x_0} \tag{C.39}$$

As derived in Appendix A. Comparing (C.39) with (C.31) shows that

$$k_1 = \gamma M \tag{C.40}$$

so that in (C.37) this gives

$$U_\tau = \frac{c^2}{2}\left(1-\frac{2\gamma M}{c^2\sigma}\right) \tag{C.41}$$

and via the same process as at (C.32) shows that

$$U_\tau = \frac{\upsilon_\tau^2}{2} \tag{C.42}$$

and thus from (C.41) and (C.42)

$$\boldsymbol{\upsilon_\tau} = c\left(1-\frac{2\gamma M}{c^2\sigma}\right)^{1/2}\boldsymbol{x_0} = cu\boldsymbol{x_0} \tag{C.43}$$

as derived in [11] and shown in Appendix A.

(iv) Case 4 - Static, Temporal, Internal.
Here (C.29) becomes

$$\nabla_4\cdot\boldsymbol{D_{i\tau}} = \nabla_4\cdot\left(\frac{u_i}{4\pi\gamma\left(1-u_i^2\right)^{1/2}}\nabla_4 U_{i\tau}\boldsymbol{x_0}\right) = \rho_v \tag{C.44}$$

Again because $|\boldsymbol{D}_{i\tau}| = |\boldsymbol{D}_{i\sigma}|$, (C.44) can be expanded to

$$\frac{\partial}{\partial\sigma_i}\left(\frac{\sigma_i^2 u_i}{\left(1-u_i^2\right)^{1/2}}\nabla_4 U_{i\tau}\right) = 4\pi\gamma\rho_v = \frac{3\gamma M}{\sigma_g^3} \tag{C.45}$$

The first integral of (C.45) is

$$\nabla_4 U_{i\tau} = \left(\frac{\gamma M\sigma_i}{\sigma_g^3}+\frac{k_1}{\sigma_i^2}\right)\frac{\left(1-u_i^2\right)^{1/2}}{u_i}\boldsymbol{x_0} = \boldsymbol{A_{i\tau}} \tag{C.46}$$

$\boldsymbol{A_{i\tau}}$ is the temporal Acceleration Potential vector internal to the source at σ_i and lies only along the $\boldsymbol{x}_0$ axis. The gradient of $U_{i\tau}$ can therefore be written

$$\frac{\partial U_{i\tau}}{\partial x_0} = \left(\frac{\gamma M\sigma_i}{\sigma_g^3}+\frac{k_1}{\sigma_i^2}\right)\frac{\left(1-u_i^2\right)^{1/2}}{u_i}\boldsymbol{x_0} \tag{C.47}$$

and this becomes

$$\frac{1}{cu_i}\frac{\partial U_{i\tau}}{\partial\sigma_i}\frac{\partial\sigma_i}{\partial\tau} = \left(\frac{\gamma M\sigma_i}{\sigma_g^3}+\frac{k_1}{\sigma_i^2}\right)\frac{\left(1-u_i^2\right)^{1/2}}{u_i}\boldsymbol{x_0} \tag{C.48}$$

Inserting (C.25) for $\partial\sigma_i/\partial\tau$ this becomes

$$\frac{\partial U_{i\tau}}{\partial\sigma_i} = \left(\frac{\gamma M\sigma_i}{\sigma_g^3}+\frac{k_1}{\sigma_i^2}\right)\boldsymbol{x_0} \tag{C.49}$$

Integrating (C.49) then gives

$$U_{i\tau} = \left(\frac{\gamma M \sigma_i^2}{2\sigma_g^3} - \frac{k_1}{\sigma_i} + k_2 \right) \tag{C.50}$$

When $\sigma_i = \sigma_g$, from (C.41)

$$U_{i\tau} = U_\tau = \frac{c^2}{2} - \frac{\gamma M}{\sigma_g} \tag{C.51}$$

so that in (C.50) this gives k_2 as

$$k_2 = \frac{c^2}{2} - \frac{3\gamma M}{2\sigma_g} + \frac{k_1}{\sigma_g} \tag{C.52}$$

which substituted back into (C.50) yields

$$U_{i\tau} = \frac{\gamma M \sigma_i^2}{2\sigma_g^3} - \frac{k_1}{\sigma_i} + \frac{c^2}{2} - \frac{3\gamma M}{2\sigma_g} + \frac{k_1}{\sigma_g} \tag{C.53}$$

Now applying Gauss' law to $\boldsymbol{D_{i\tau}}$

$$\oiint \boldsymbol{D_{i\tau}} \cdot d\boldsymbol{s} = \frac{u_i}{\left(1 - u_i^2\right)^{1/2}} \frac{\boldsymbol{A_{i\tau}}}{4\pi\gamma} \oiint d\boldsymbol{s_i} = M_i = M\frac{\sigma_i^3}{\sigma_g^3} \tag{C.54}$$

and from this

$$\boldsymbol{A_{i\tau}} = \frac{\left(1 - u_i^2\right)^{1/2}}{u_i} \frac{\gamma M \sigma_i}{\sigma_g^3} \boldsymbol{x_0} \tag{C.55}$$

as shown in Appendix A. Comparing this to (C.46) shows that $k_1 = 0$, so that in (C.53) this yields

$$U_{i\tau} = \frac{c^2}{2} \left(1 - \frac{3\gamma M}{c^2 \sigma_g} + \frac{\gamma M \sigma_i^2}{c^2 \sigma_g^3} \right) \tag{C.56}$$

Finally, via the usual process, with

$$U_{i\tau} = \frac{\upsilon_{i\tau}^2}{2} \tag{C.57}$$

then

$$\boldsymbol{\upsilon_{i\tau}} = c \left(1 - \frac{3\gamma M}{c^2 \sigma_g} + \frac{\gamma M \sigma_i^2}{c^2 \sigma_g^3} \right)^{1/2} \boldsymbol{x_0} \tag{C.58}$$

as derived in [11] and stated in Appendix A.

(v) The Steady State Case.

Solutions to the steady state case are not presented here because neither the translational nor the rotational motions of the source contribute directly to the generation of gravity. Secondary effects only exist and are as follows.

(a) Both motions will cause a relativistic increase in the mass of the source which results in a small increase in the magnitude of the gravitational effect.

(b) The relativistic mass increase due to the rotational motion will result in a very small variation of the gravitational field in the azimuthal direction. This will have some effect on the orbital dynamics of orbiting bodies.

(c) Both motions enter into the temporal velocity equation as extra boundary conditions but, as stated above, do not affect the generation of gravity.

APPENDIX D.

Gravitational Waves.

With the purported absence of the gravitational equivalent of $\boldsymbol{B}$, the magnetic flux density of electromagnetics, in the Relativistic Domain theory of gravitation, the velocity of propogation of gravitational changes cannot be determined via the gravitational Maxwell equations, and thereby cannot be equated to the velocity of light, in the same manner as for electromagnetic waves. However, in this theory, it is quite clear that the velocity of propogation of such changes must be $\upsilon_{i\sigma}$ and υ_{σ}, the spatial expansion velocity generated by the source. Reference to (C.14) and (C.25) clearly shows that any variation to the mass, M, of the source, or to σ, the distance from it, results is a variation of $\upsilon_{i\sigma}$ and thereby υ_{σ} which in turn carries with it a variation in the Acceleration Potential.

D.1 Variation of Mass - An Example.
The time for a gravitational shock wave, caused by a sudden change in the mass of a gravitational source, to travel a distance σ is given by

$$\tau = \left(\frac{2\sigma^3}{9\gamma M}\right)^{1/2} \tag{D.1}$$

Now consider the Crab Nebula, the result of a large supernova observed by Chinese and Japanese astronomers in 1054A.D. If this star were typical of the largest, with a radius of 1.4E14cm. and a mass of $\sim$ 2E36gms, then the time taken for the leading edge of the supernova gravitational shock wave to reach Earth would be, from (D.1), (σ = 5500LY).

$$\tau \cong 4.84E17 \; years \tag{D.2}$$

As the actual event took place only some 6,500 years ago, clearly the shock wave will not reach Earth until long after the latter's demise.

D.2 Variation in Distance - An Example.
Assume that there is a large binary system sufficiently close such that its gravitational waves have already reached Earth. The question is whether the wave could be detected. If the system were one of a large planet or small star orbiting a very large star, then the composite temporal rate generated would be

$$u_B = \left\{1 - \frac{2\alpha_S}{\sigma_S} - \frac{2\alpha_P}{(\sigma_S + \sigma_P \sin\omega_P\tau)}\right\}^{1/2} \tag{D.3}$$

where

u_B is the composite temporal rate of the binary system.

α_S is the gravitational radius of the large star.

σ_S is the distance of the large star from the measuring point, (Earth).

α_P is the gravitational radius of the orbiting planet.

σ_P is the average orbital radius of the orbiting planet.

ω_P is the average angular rate of the orbiting planet.

then taking the spatial differential of (D.3)

$$\frac{du_B}{d\sigma} = \frac{\dfrac{\alpha_P}{(\sigma_S + \sigma_P \sin \omega_P \tau)} + \dfrac{\alpha_S}{\sigma_S^2}}{\left(1 - \dfrac{2\alpha_S}{\sigma_S} - \dfrac{2\alpha_P}{(\sigma_S + \sigma_P \sin \omega_P \tau)}\right)^{1/2}} \tag{D.4}$$

and therefore the force generated on a test mass m_t, at the measuring point, is given by

$$F = -m_t c^2 u_B \frac{du_B}{d\sigma} = -\frac{\gamma m_t M_S}{\sigma_S^2}\left\{1 + \frac{M_P}{M_S}\left(1 + \frac{\sigma_P}{\sigma_S}\sin \omega_P \tau\right)^{-2}\right\} \tag{D.5}$$

where

M_S is the mass of the large star.

M_P is the mass of the planet.

Expanding the inverse square

$$F = -\frac{\gamma m_t M_S}{\sigma_S^2}\left\{1 + \frac{M_P}{M_S}\left(1 - \frac{2\sigma_P}{\sigma_S}\sin \omega_P \tau + \frac{3\sigma_P^2}{\sigma_S^2}\sin^2 \omega_P \tau - \cdots\right)\right\} \tag{D.6}$$

So that the fundamental component of variation is inversely proportional to the cube of the distance to the large star, while the period of variation is directly proportional to the orbital angular rate of the orbiting planet.

In view of the above two examples, it is considered unlikely that gravitational waves will ever be detected on Earth, unless they emanated from a catastrophic event that occurred astronomically close by.

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